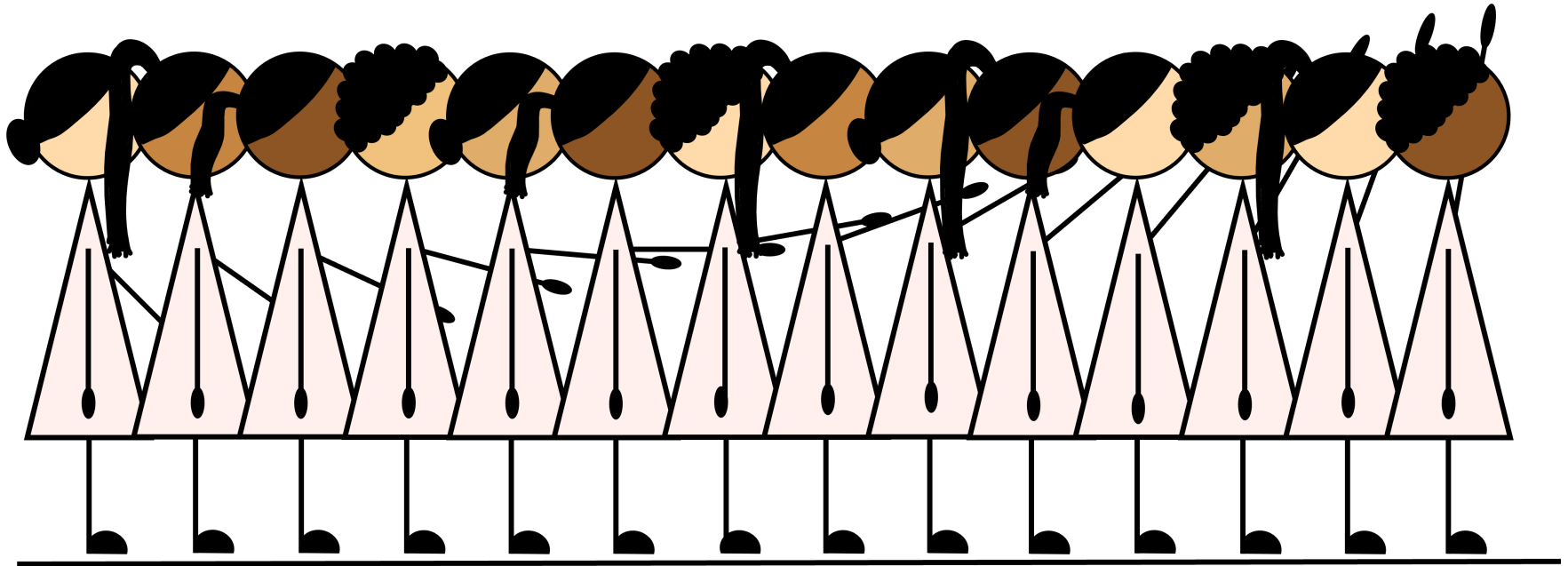


Kinesthetic Activities for Learning Quantum Mechanics

Elizabeth Gire

Oregon State University
AAPT Summer 2023



Coauthors



Corinne Manogue
she/they



Kelby Hahn
they/she



Adam Frye
he/him

osuper.science.oregonstate.edu

Faculty

Elizabeth Gire
Corinne Manogue
Doris Li
Patty Hamerski
Tevian Dray
Emily Van Zee
Paul Emigh

Grad Students

Christian Solorio
Dustin Treece
Adam Frye
Jason Ward

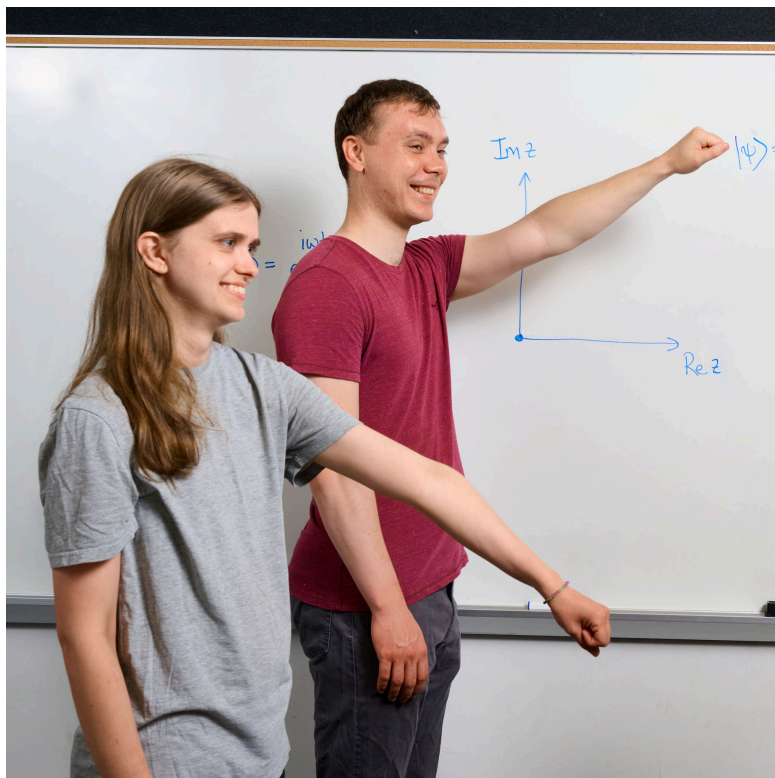
Former Members

Jonathan Alfson
Kelby Hahn
David Roundy
Michael Vignal
MacKenzie Lenz
Greg Mulder
Emily Smith
Len Cerny
Kerry Brown
Grant Sherer
Ian Founds
Mesa Walker
Mary Bridget Kustusch
Rabindra Bajracharya

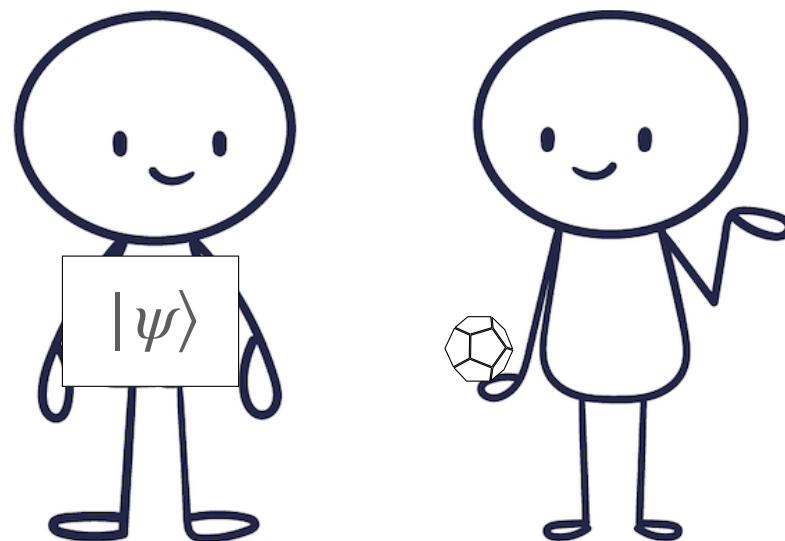


NSF DUE Grant Nos. 9653250, 0231194,
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1323800, 1612480, 1836603, 1836604

Arms Representation of Quantum States



Quantum Measurement Skit



Kinesthetic Activities for *Upper Division Quantum Mechanics?!*

Activate sensorimotor brain systems

Make decisions about how configure and move sequentially

Re-representation

For quantum systems (>1 people), have to socially negotiate

Introduces silliness and laughter

Formative assessment

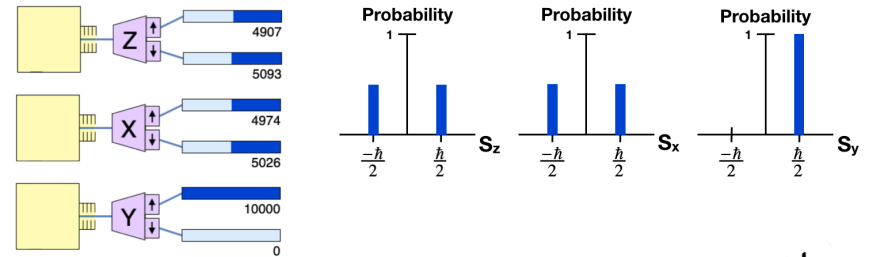
Solomon, et al., *Phys. Ed.*, 1991
Kontra, et al., *Psychol. Sci.*, 2015
Duijzer, et al., *Educ. Psychol. Rev.*, 2019
Struck & Yerrick, *J. Sci Educ. Technol.*, 2010,
Beichner, et al., *Am. J. Phys.*, 1990
Hubber , Titler, & Haslam, *Res. Sci. Educ.*, 2010

Instructional Context

Paradigms in Physics

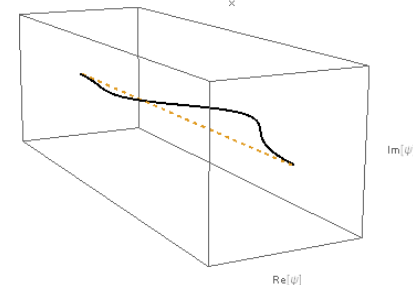
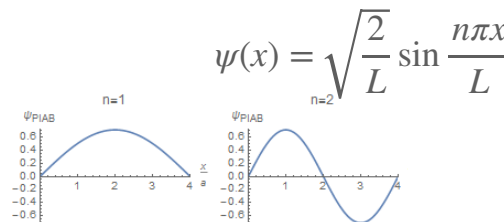
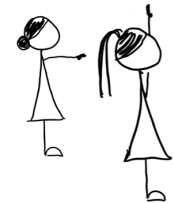
Quantum Fundamentals & Central Forces Courses

- “Spins First” Approach (McIntyre textbook)
- Stern-Gerlach Simulation to explore postulates of quantum mechanics
- Emphasize Multiple Representations
- Computational lab



$$\begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix}$$

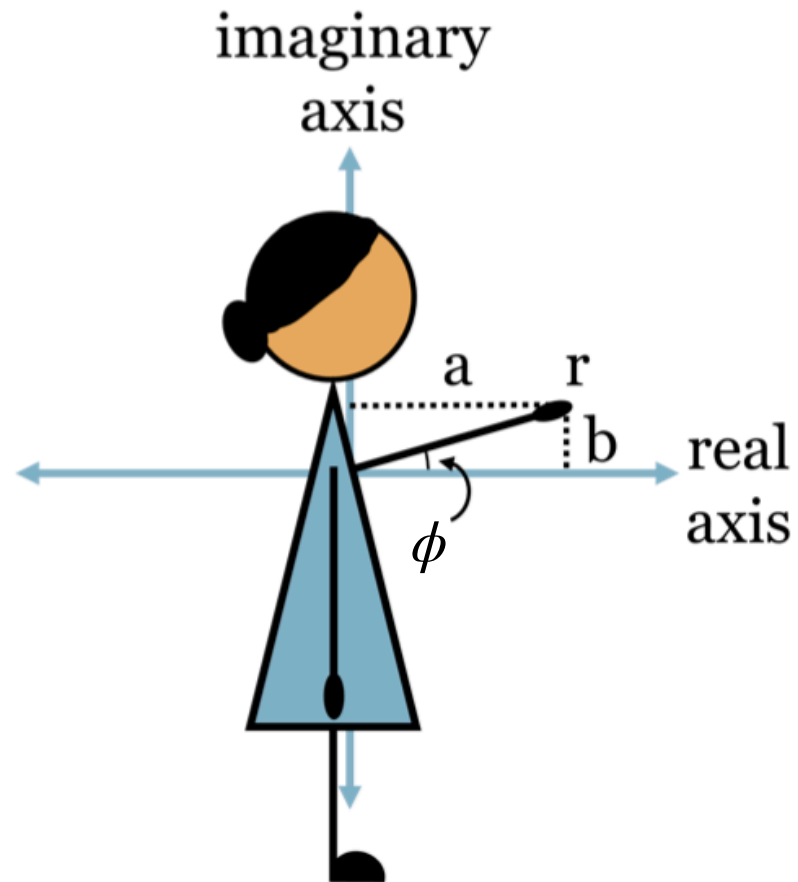
$$\frac{1}{\sqrt{2}}|+\rangle + \frac{i}{\sqrt{2}}|-\rangle$$



Emigh, et al., *Phys. Rev. PER*, 2020
 Manogue, et al., *Am. J. Phys.*, 2001

Arms Representation

Arms Basics



Arms Pros & Cons

- ✓ 4D
- ✓ Phase Angle Salient
- ✓ Accommodate Physical Ability
- ✓ Components of complex numbers vs. quantum basis
- ✓ Memorable

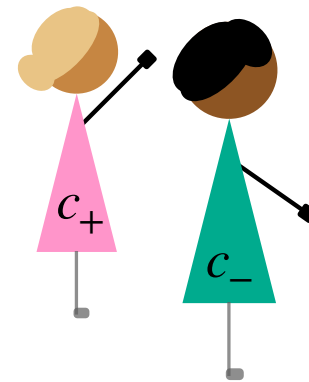
Arms Pros & Cons

- ✓ 4D
- ✓ Phase Angle Salient
- ✓ Accommodate Physical Ability
- ✓ Components of complex numbers vs. quantum basis
- ✓ Memorable
- Arm length not adjustable for different norms
- Lots of information that is not externalized
- Visualization?
- Self Consciousness

Quantum Concepts & Representations

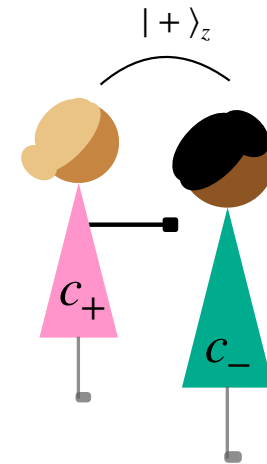
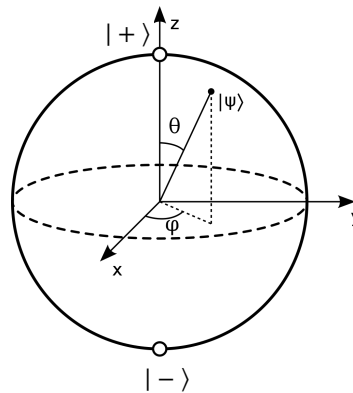
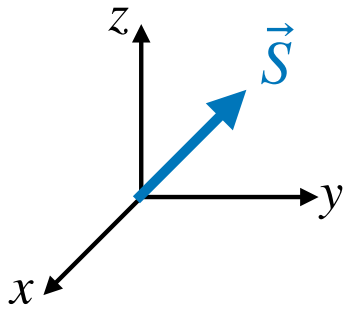
Quantum states are **vectors** with complex components

$$|\psi\rangle = c_+|+\rangle + c_-|-\rangle \quad |\psi\rangle \doteq \begin{bmatrix} c_+ \\ c_- \end{bmatrix}$$



Quantum Concepts & Representations

Cartesian space and Hilbert space are different



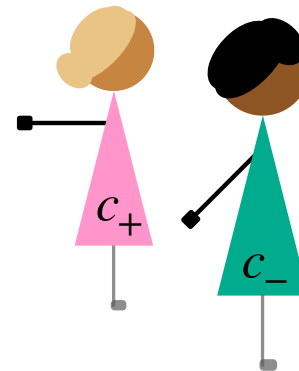
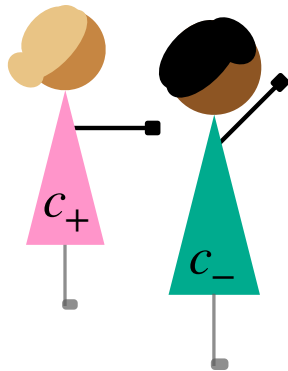
$$|\psi\rangle = \cos \frac{\theta}{2} |+\rangle + \sin \frac{\theta}{2} e^{i\varphi} |-\rangle$$

Quantum Concepts & Representations

Vectors that differ by an **overall phase** represent the same quantum state

$$|\psi\rangle = c_+|+\rangle + c_-|-\rangle$$

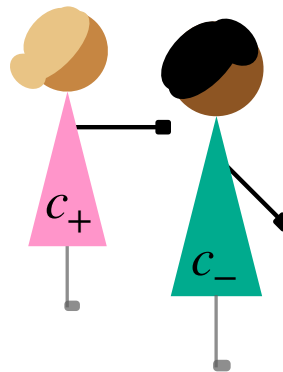
$$|\psi\rangle = e^{i\phi}(c_+|+\rangle + c_-|-\rangle)$$



Quantum Concepts & Representations

Quantum states evolve with time - **time & energy-dependent phase** on terms in energy eigenstate expansion

$$|\psi(t)\rangle = c_+ e^{-iE_+t/\hbar} |+\rangle + c_- e^{-iE_-t/\hbar} |-\rangle$$



Quantum Concepts & Representations

Formalisms for **discrete and continuous** quantum systems are related.

$$c_{\pm} = {}_z\langle \pm | \psi \rangle$$

$$\psi(x) = \langle x | \psi \rangle$$



Arms Activities

Complex Numbers

Quantum State

Relative & Overall Phase

Time Evolution

Wavefunction

Inner Product of Spin-1/2 States

Time Evolution of a Particle on a Ring

Hahn & Gire, *Am. J. Phys*, 2022

This talk

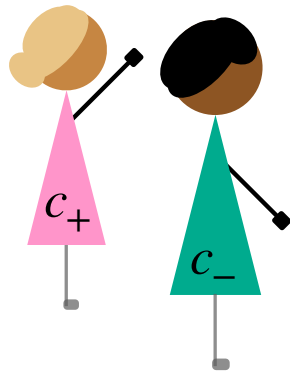
Inner Product of Spin-1/2 System

Quantum Concepts & Representations

Measurement probabilities are related to **inner products** between quantum states

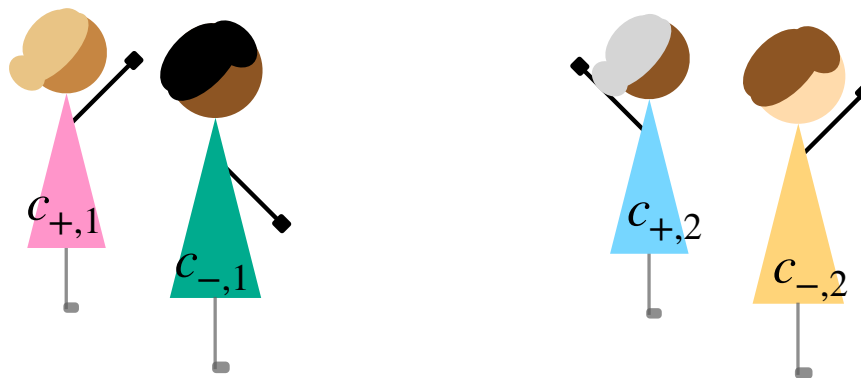
$$\mathcal{P}\left(S_z = \frac{+\hbar}{2}\right) = \left| {}_z\langle + | \psi \rangle \right|^2$$

Inner Product of Spin-1/2 System



Ask pair of students to represent
an arbitrary state.

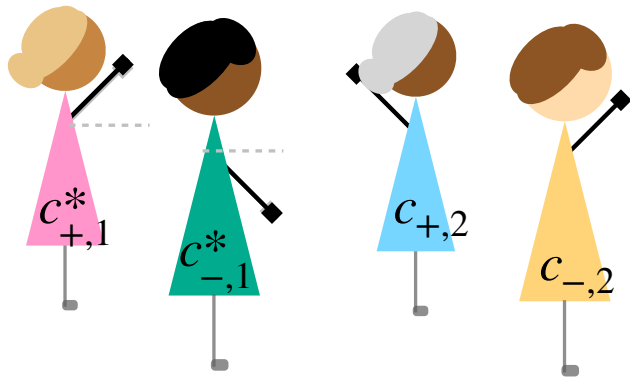
Inner Product of Spin-1/2 System



Introduce a second state (each rotated by $\pi/2$)

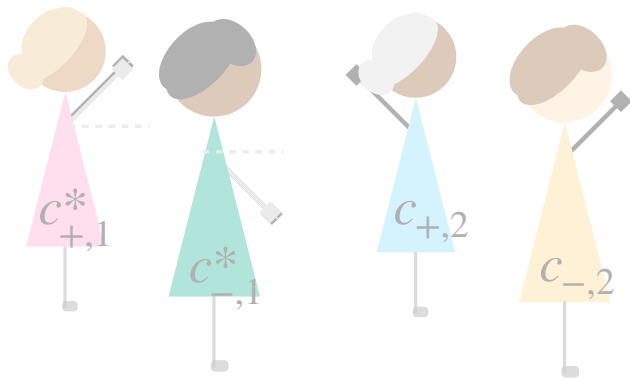
Are these states orthogonal?

Inner Product of Spin-1/2 System

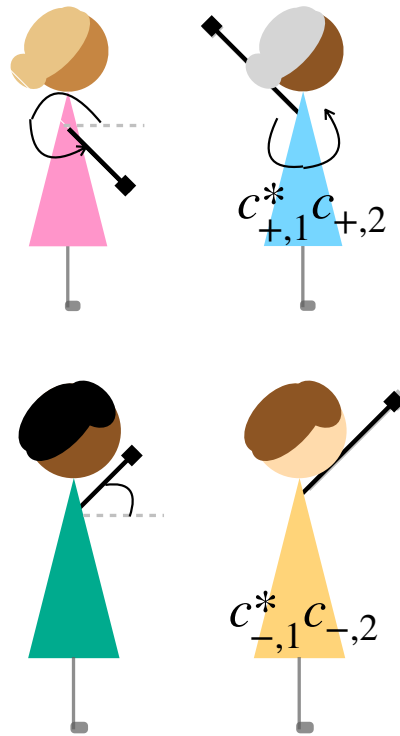


Complex Conjugate 1 pair

Inner Product of Spin-1/2 System

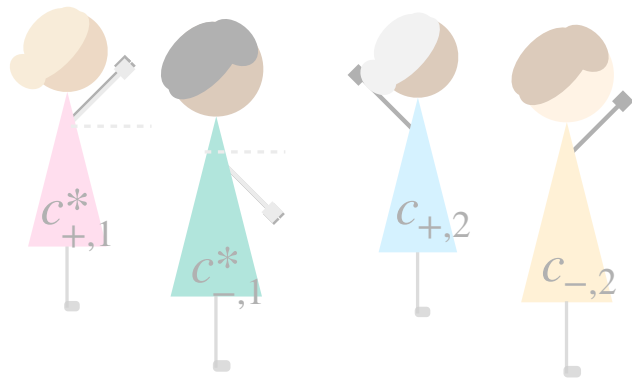


Complex Conjugate 1 pair

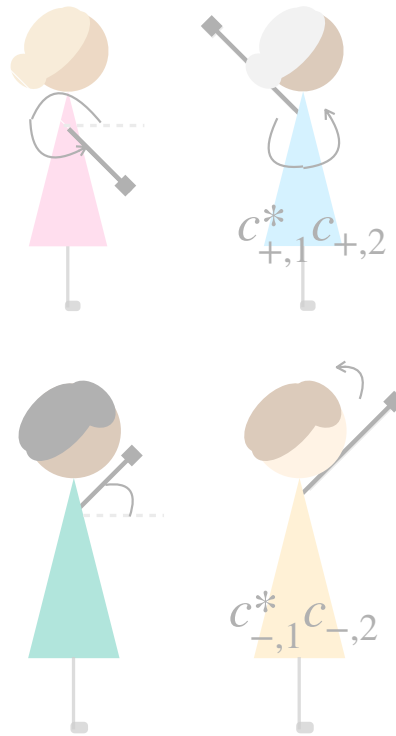


Multiply component-wise

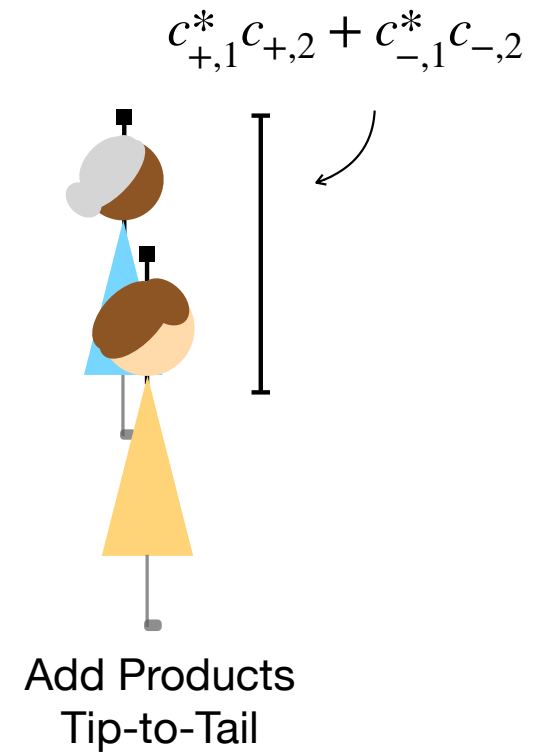
Inner Product of Spin-1/2 System



Complex Conjugate 1 pair



Multiply component-wise



Add Products
Tip-to-Tail

Pedagogical Affordances

✓ Emphasizes steps, particularly

- ▶ complex conjugate
- ▶ aligning components

– Arm length not adjustable for different norms

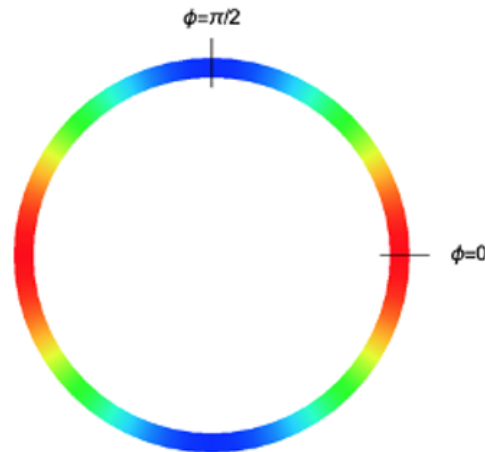
– Adding “tip-to-tail” requires effort

Time Evolution of a Quantum Particle on a Ring

Time Evolution of Particle on a Ring

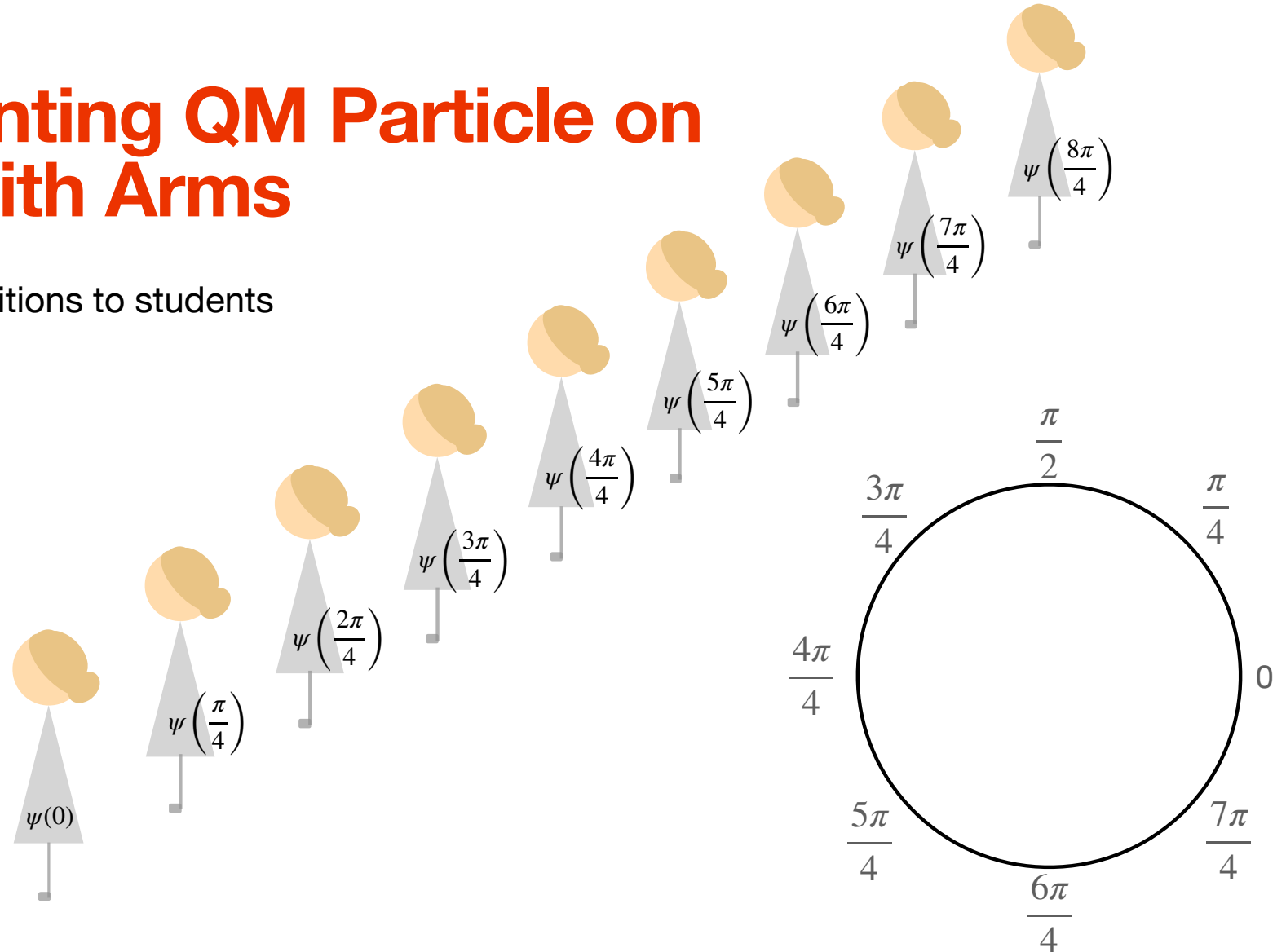
$$E_m(\phi) \doteq \langle \phi | m \rangle = \frac{1}{\sqrt{2\pi}} e^{im\phi} \qquad E_m = \frac{m^2 \hbar^2}{2I}$$

Probability Density for $m=1$



Representing QM Particle on a Ring with Arms

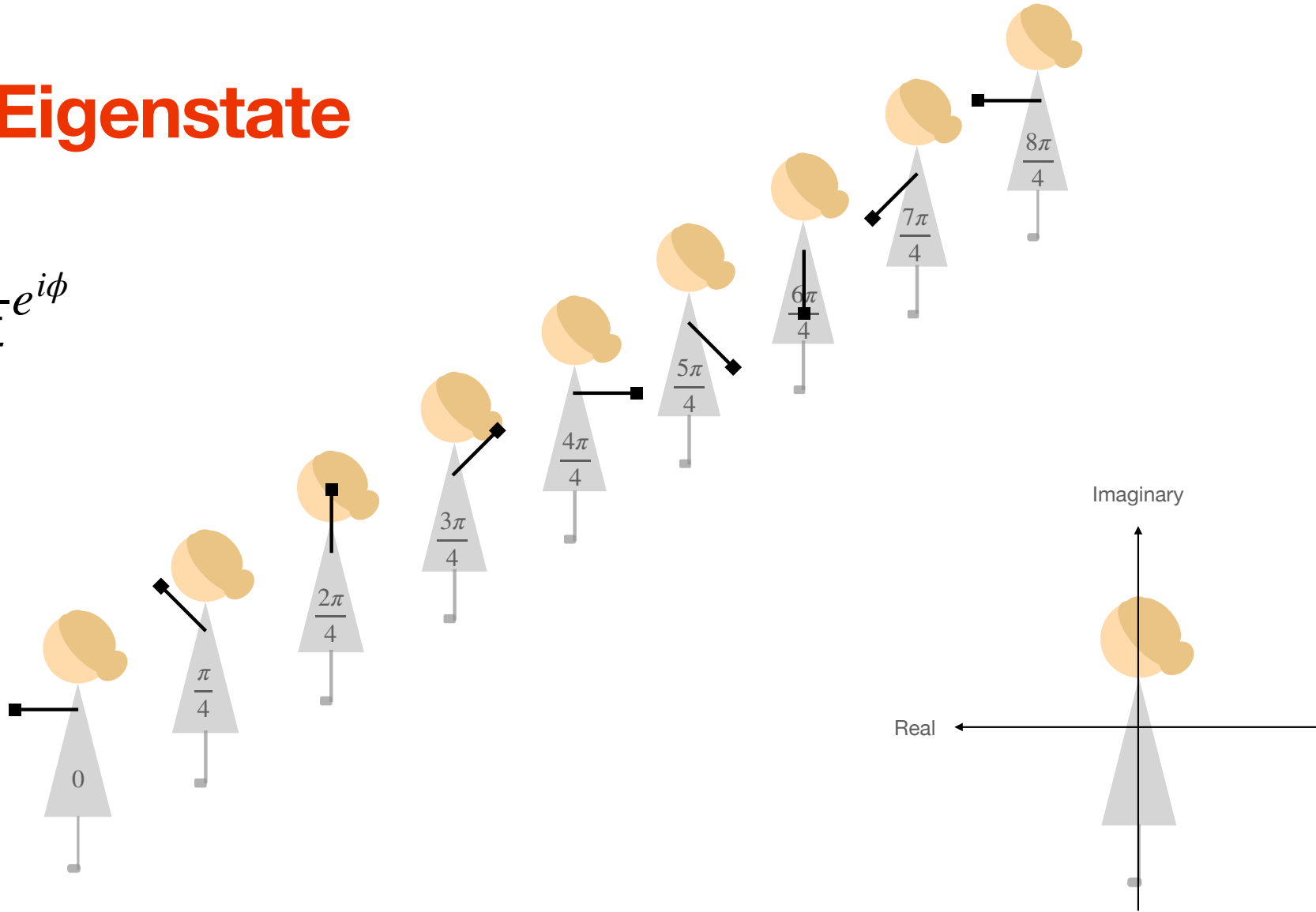
Assign angular positions to students



Energy Eigenstate

$m=1$

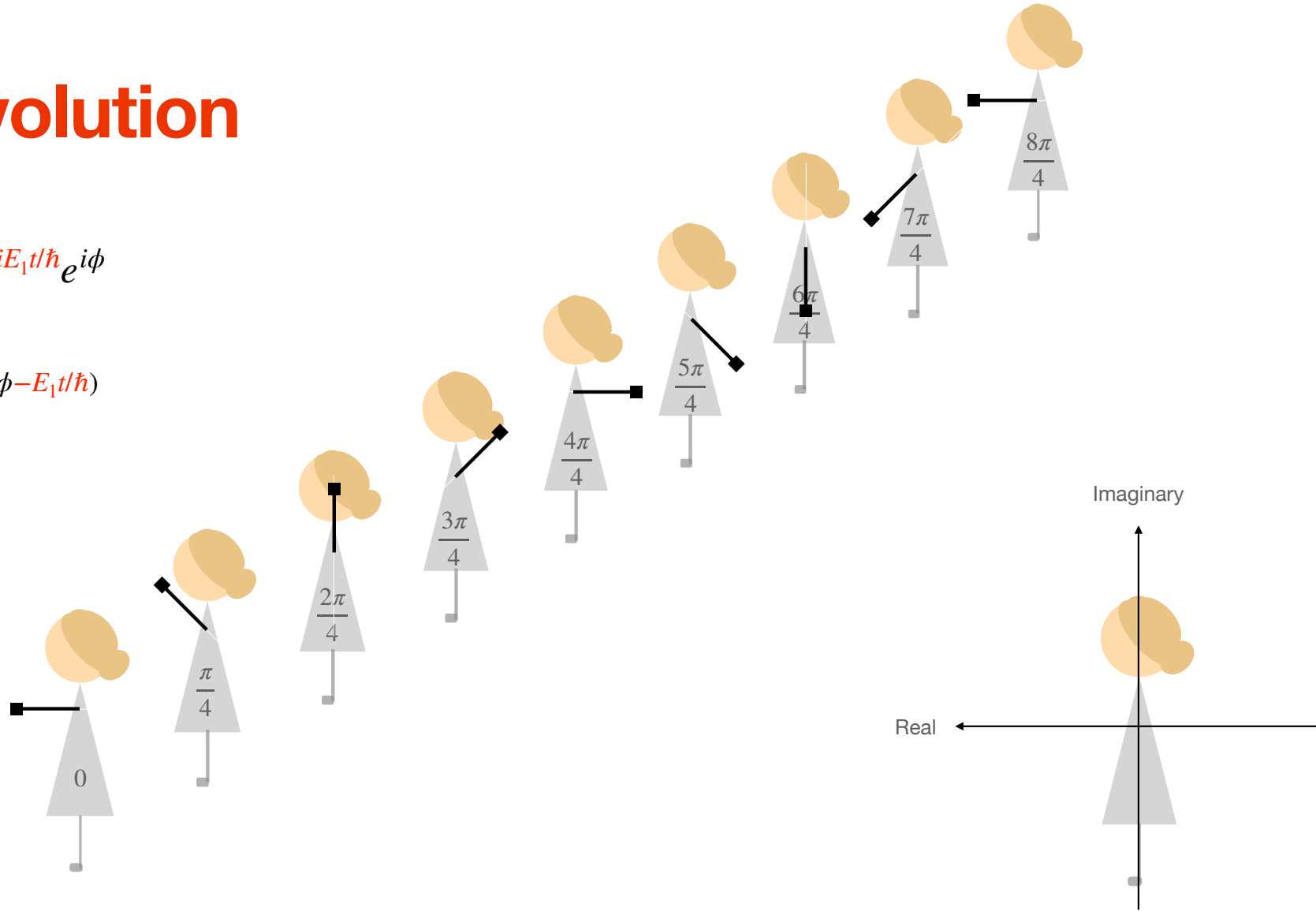
$$E_1(\phi) = \frac{1}{\sqrt{2\pi}} e^{i\phi}$$



Time Evolution

$m=1$

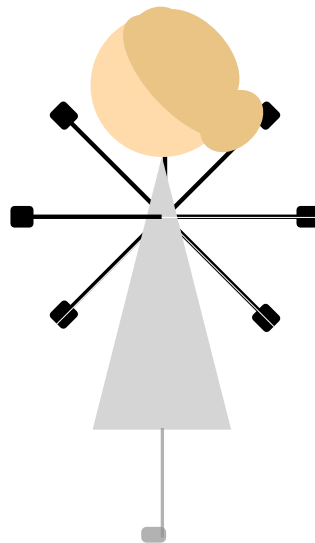
$$E_1(\phi) = \frac{1}{\sqrt{2\pi}} e^{-iE_1 t/\hbar} e^{i\phi}$$
$$= \frac{1}{\sqrt{2\pi}} e^{i(\phi - E_1 t/\hbar)}$$



Time Evolution - Shoulder View

$m=1$

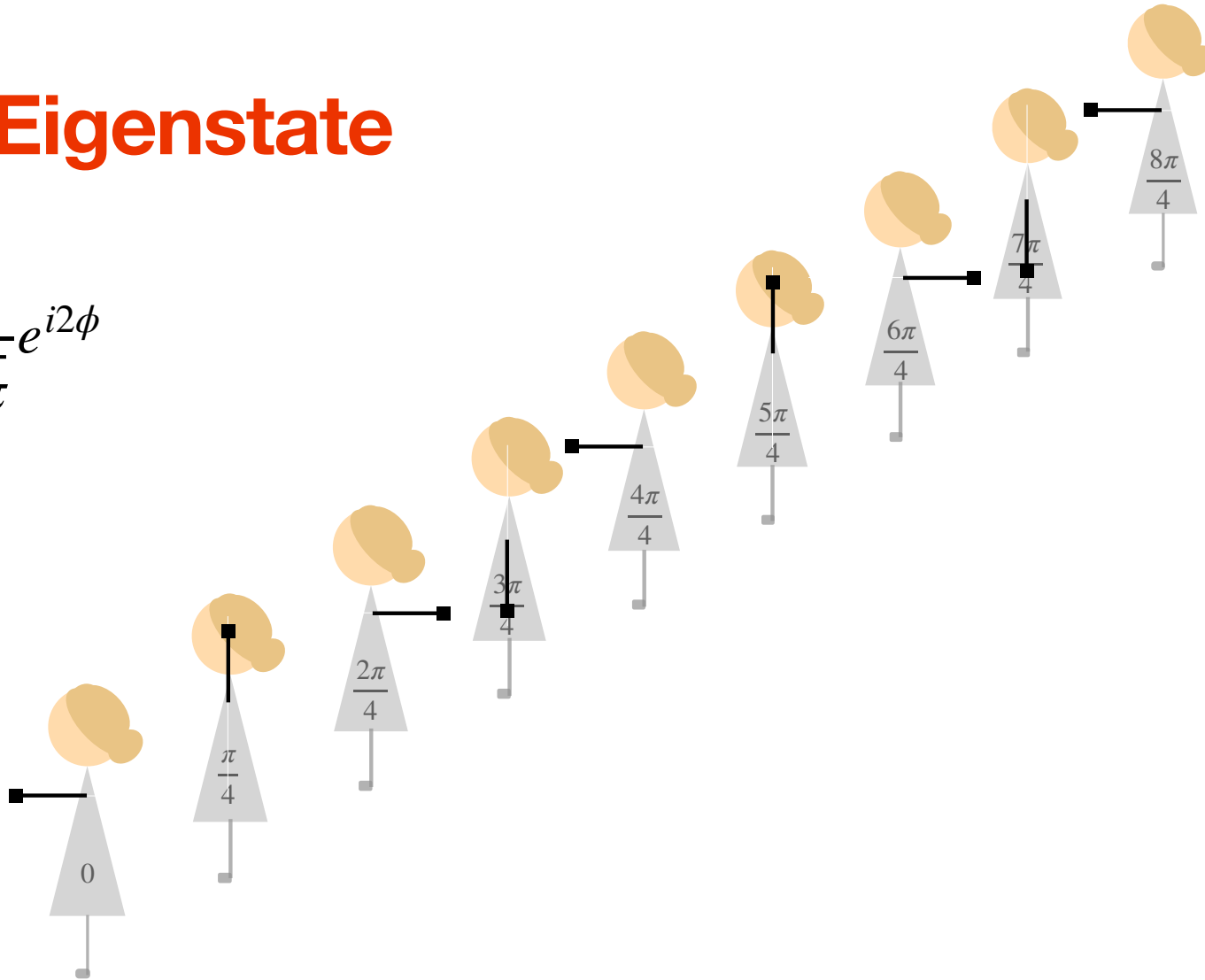
$$\begin{aligned} E_1(\phi) &= \frac{1}{\sqrt{2\pi}} e^{-iE_1 t/\hbar} e^{i\phi} \\ &= \frac{1}{\sqrt{2\pi}} e^{i(\phi - E_1 t/\hbar)} \end{aligned}$$



Energy Eigenstate

$m=2$

$$E_2(\phi) = \frac{1}{\sqrt{2\pi}} e^{i2\phi}$$

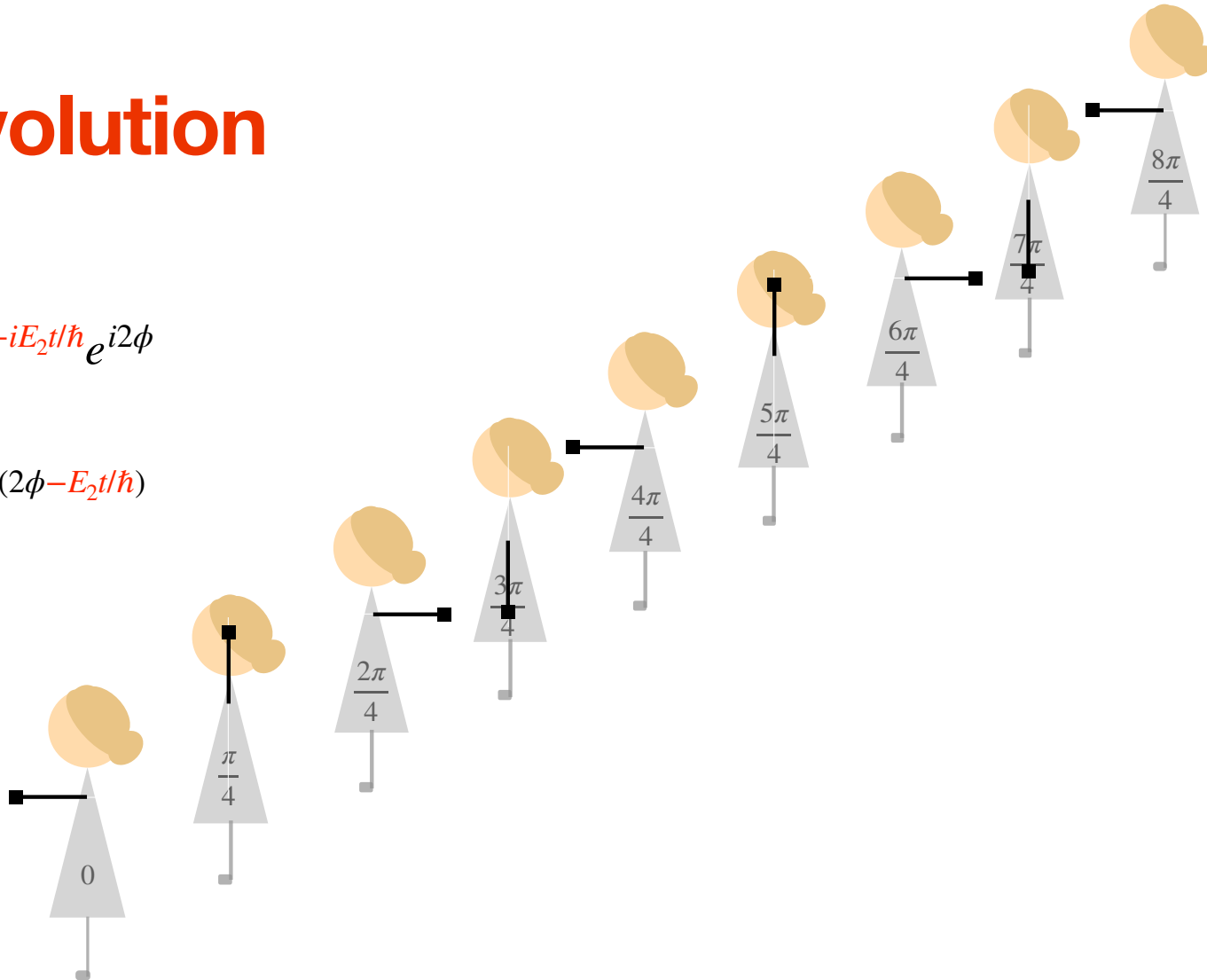


Time Evolution

$m=2$

$$E_2(\phi) = \frac{1}{\sqrt{2\pi}} e^{-iE_2 t/\hbar} e^{i2\phi}$$
$$= \frac{1}{\sqrt{2\pi}} e^{i(2\phi - E_2 t/\hbar)}$$

$$E_2 = 4E_1$$



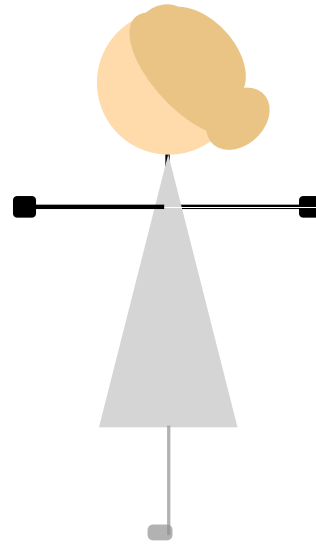
Time Evolution

$m=2$

$$E_2(\phi) = \frac{1}{\sqrt{2\pi}} e^{-iE_2 t/\hbar} e^{i2\phi}$$

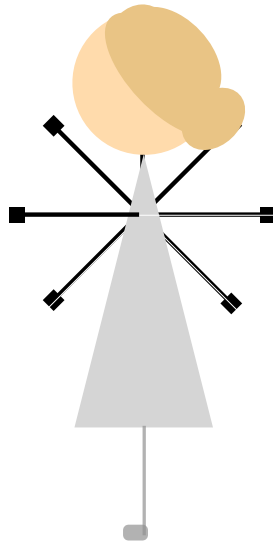
$$= \frac{1}{\sqrt{2\pi}} e^{i(2\phi - E_2 t/\hbar)}$$

$$E_2 = 4E_1$$

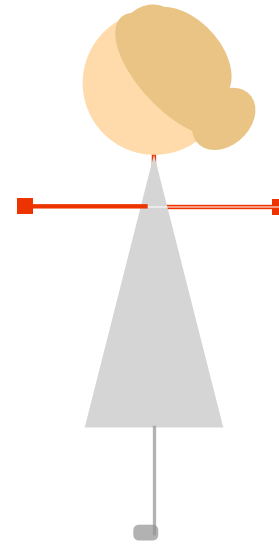


Time Evolution

m=1

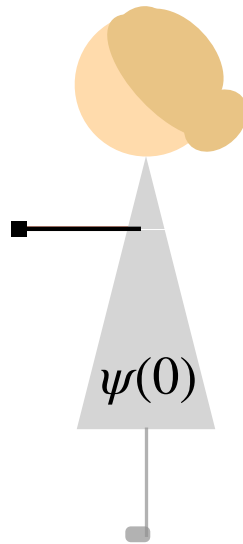


m=2



$$E_2 = 4E_1$$

Superposition



QuVis (St Andrews)

<https://www.st-andrews.ac.uk/physics/quvis/>

Simulation
Challenges
QuVis

Time-development of infinite well quantum states

Wave function: $1/\sqrt{2}(\psi_1(x)e^{-i(0\pi+1.89\pi)} + \psi_2(x)e^{-i(6\pi+1.56\pi)})$
 Time $t = (0 + 0.944) h/E_1$

Complex plane at point $x_0 = 0.25L$

$1/\sqrt{2} \psi_1(x_0)$
 $1/\sqrt{2} \psi_2(x_0)$
 $1/\sqrt{2}(\psi_1(x_0)+\psi_2(x_0))$

The time-dependence of the one-dimensional infinite square well energy eigenstate $\psi_n(x,t) = \psi_n(x) e^{-iE_n t/\hbar}$ corresponds to a rotation of $\psi_n(x)$ in the complex plane, with angular frequency $\omega_n = E_n/\hbar$ where $E_n = n^2 E_1$. Use the "?" buttons for more information. Then try the Challenges!

Probability density $|\psi|^2$

$|\psi(x_0)|^2 = 1.10 (2/L)$ (red line)

Main Controls

Show $\psi(x,t)$
Point $x_0 = 0.25L$
0.05L 0.95L
?

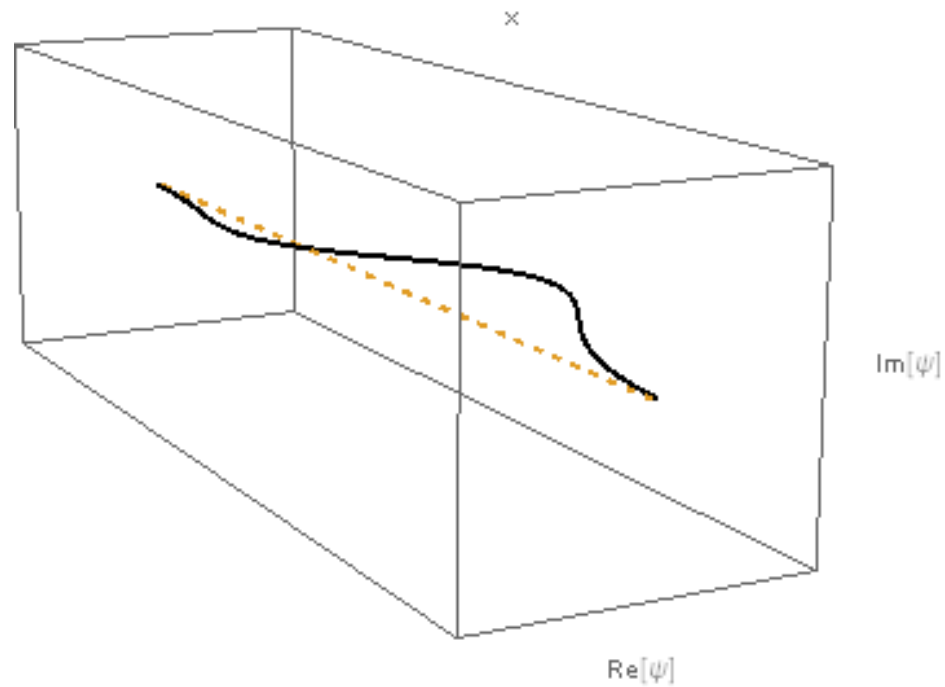
Show probability density $|\psi(x,t)|^2$ graph ?

ψ_1
 ψ_2
 $1/\sqrt{2}(\psi_1+\psi_2)$ Quantum state ?

Reset to $t=0$
Resume
Time step back
Time step forward

Graphical Superposition

Infinite Square Well



Pedagogical Affordances

- ✓ For eigenstates, arms are norm=1
- ✓ Highlights differences between stationary and non-stationary states
- ✓ Superposition at each position results in complicated time evolution
- Requires at least 8 students
- Completing the superposition is difficult

Quantum Measurement Skit

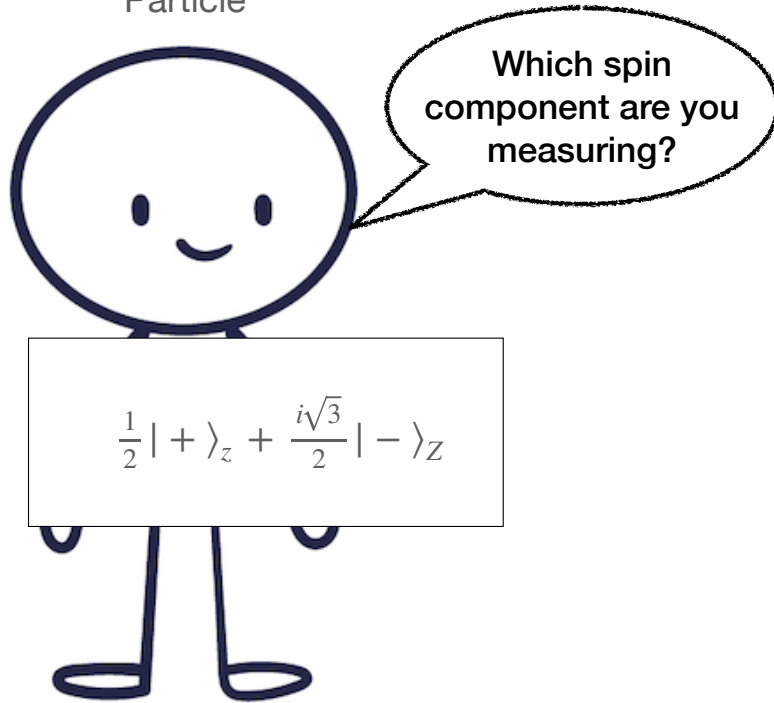
Quantum Concepts & Representations

Measurement results in a **probabilistic projection** onto the output basis **and renormalization**

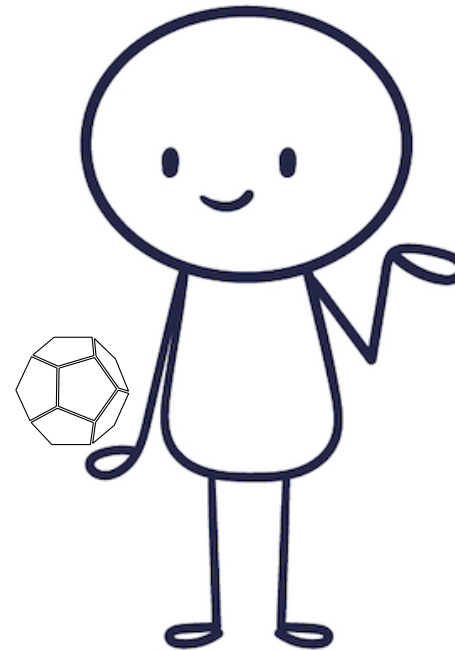
$$|\psi_{out}\rangle = \frac{\hat{P}|\psi_{in}\rangle}{\langle\psi_{in}|\hat{P}|\psi_{in}\rangle}$$

Quantum Measurement Skit

Particle

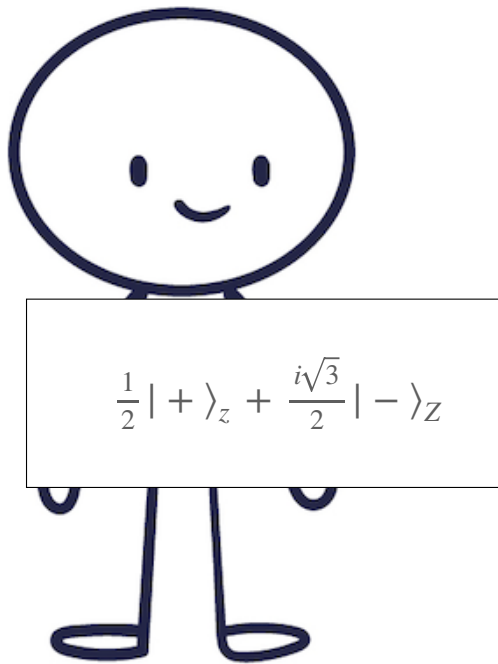


Stern-Gerlach Apparatus

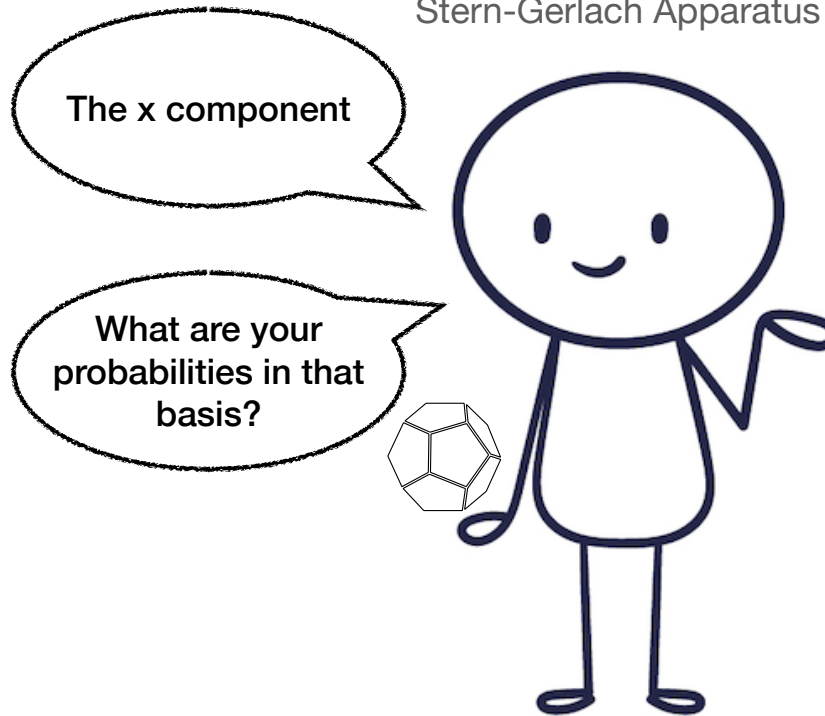


Quantum Measurement Skit

Particle

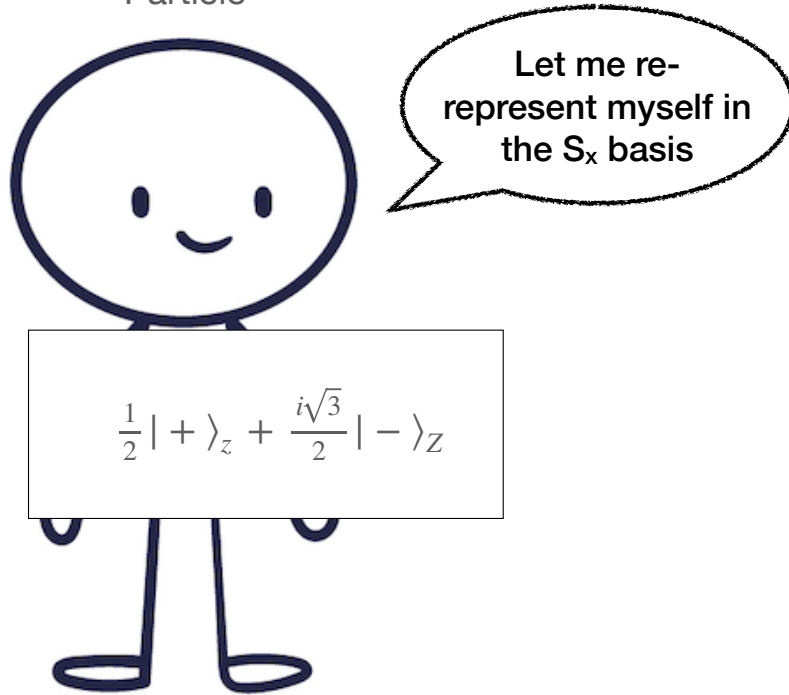


Stern-Gerlach Apparatus

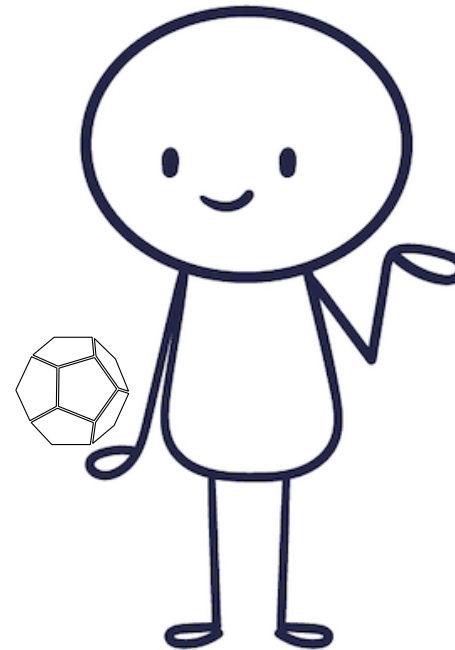


Quantum Measurement Skit

Particle

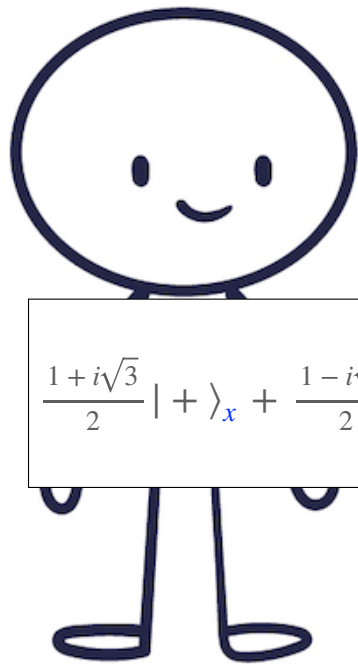


Stern-Gerlach Apparatus



Quantum Measurement Skit

Particle

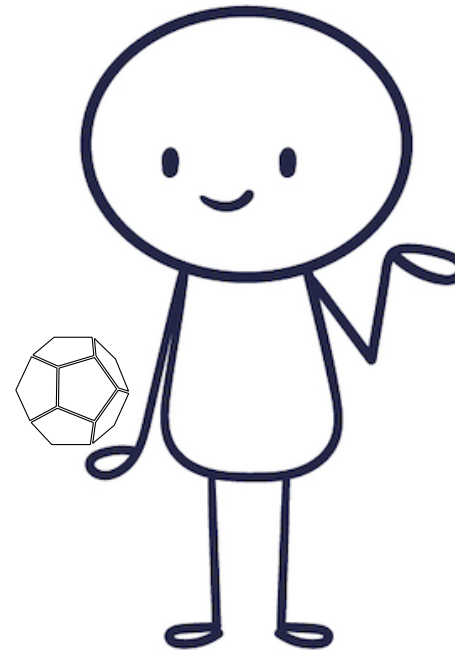


Let me re-
represent myself in
the S_x basis

Oh - the
probabilities are 1/2
and 1/2

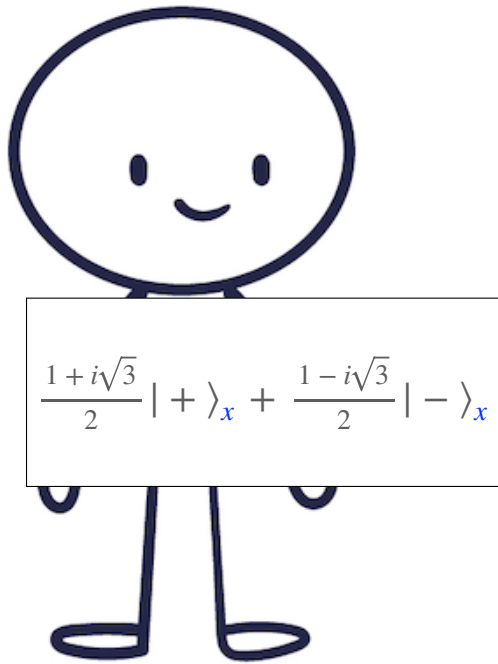
$$\frac{1+i\sqrt{3}}{2} |+\rangle_x + \frac{1-i\sqrt{3}}{2} |-\rangle_x$$

Stern-Gerlach Apparatus



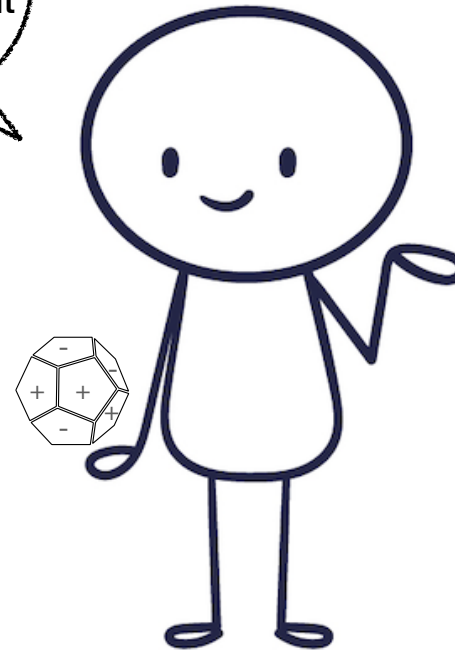
Quantum Measurement Skit

Particle



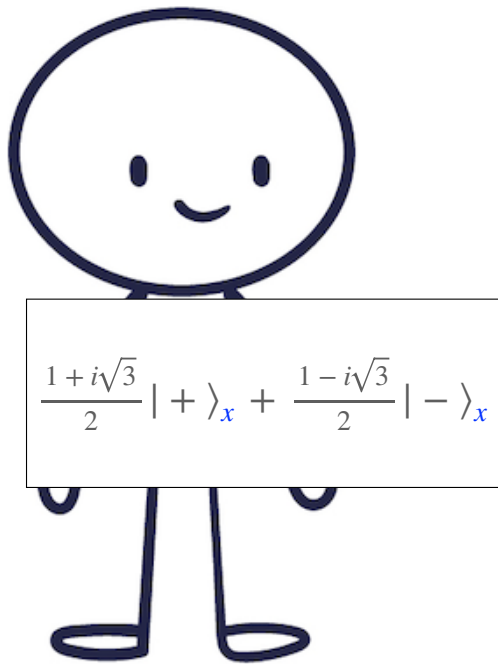
Great!
Let me weight my
die to match so we
can see what the result
of the measurement
is!

Stern-Gerlach Apparatus



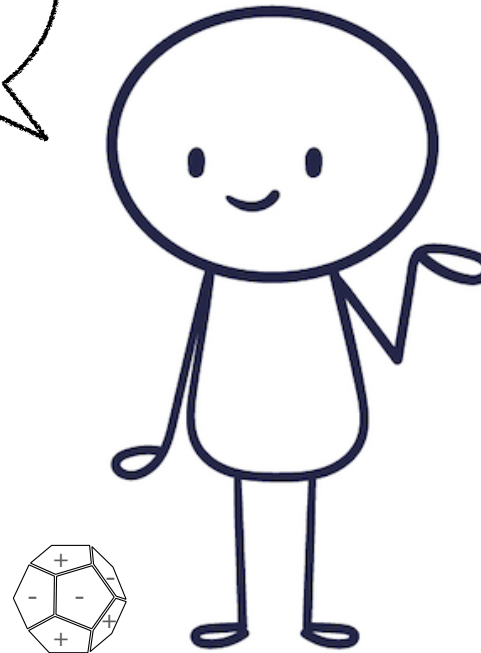
Quantum Measurement Skit

Particle



Oh, look! Your x-component of spin is now $+\hbar/2$

Stern-Gerlach Apparatus



Quantum Measurement Skit

Particle



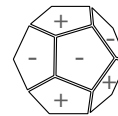
$$\frac{1+i\sqrt{3}}{2} |+\rangle_x + \frac{1-i\sqrt{3}}{2} |-\rangle_x$$



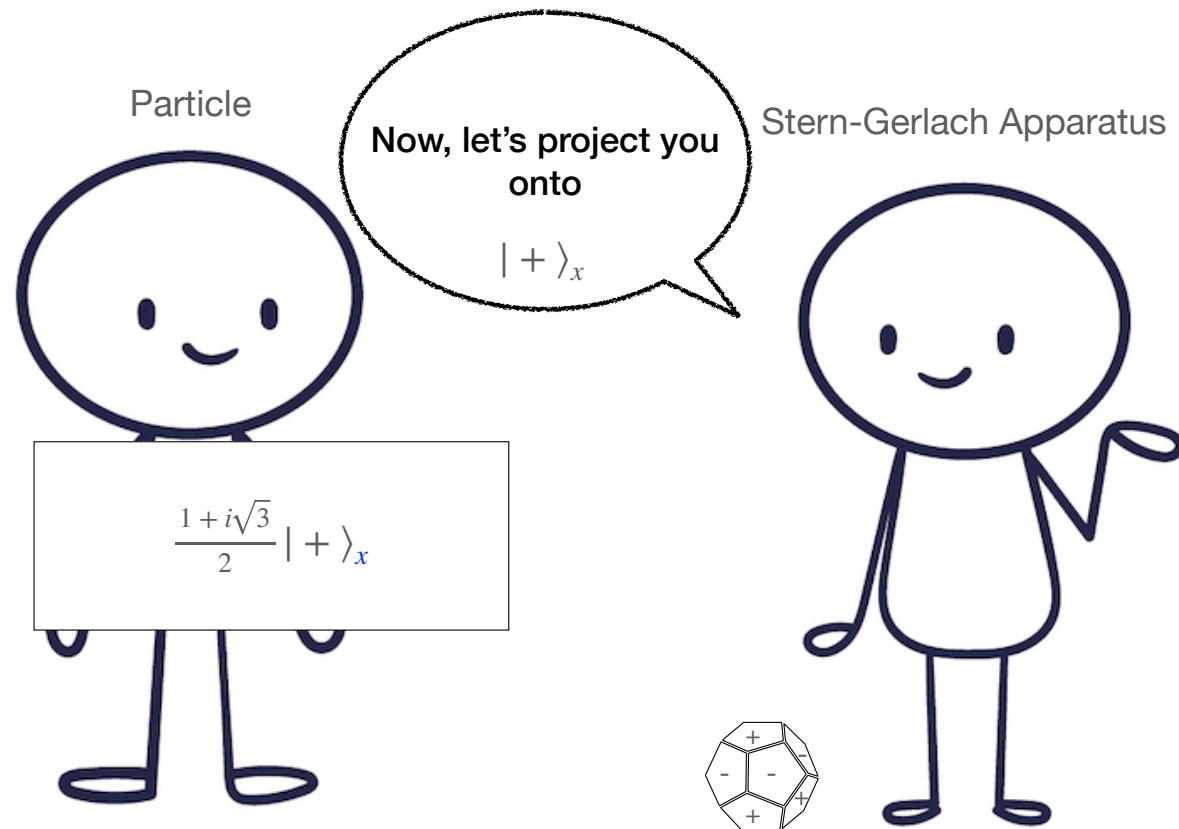
Now, let's project you
onto

$$|+\rangle_x$$

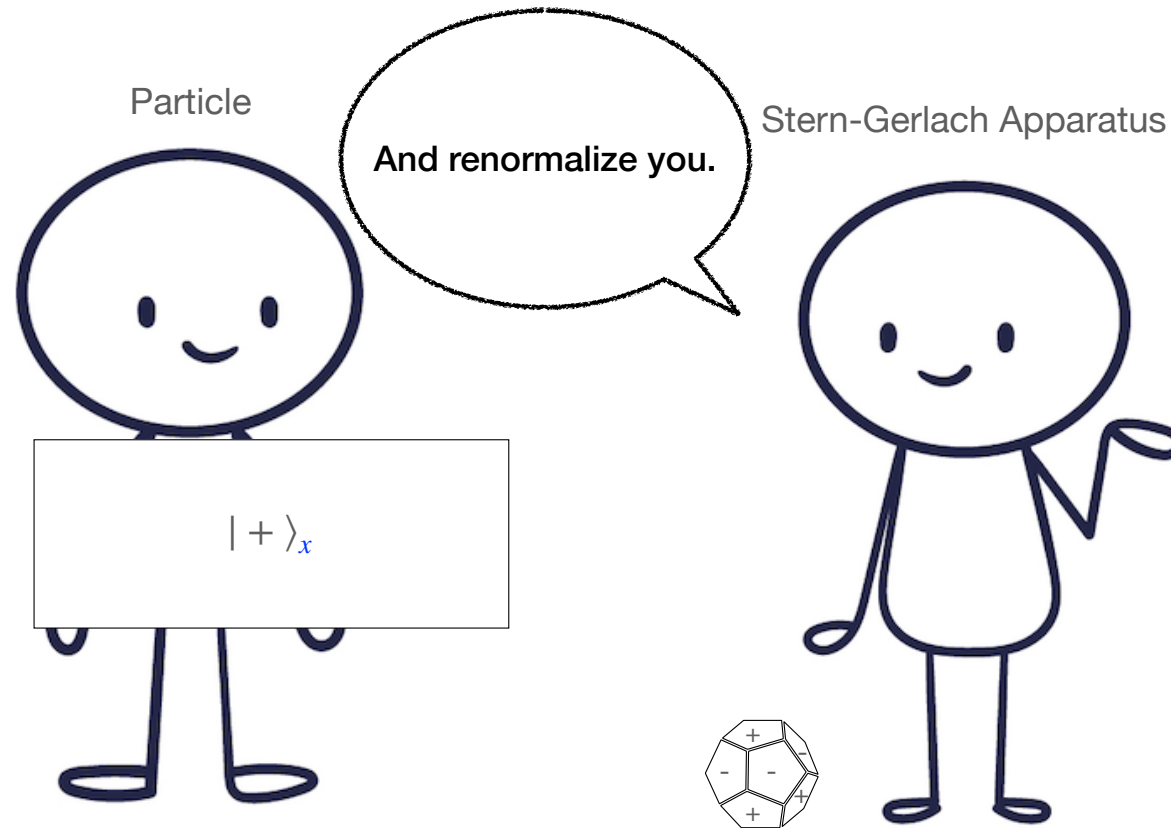
Stern-Gerlach Apparatus



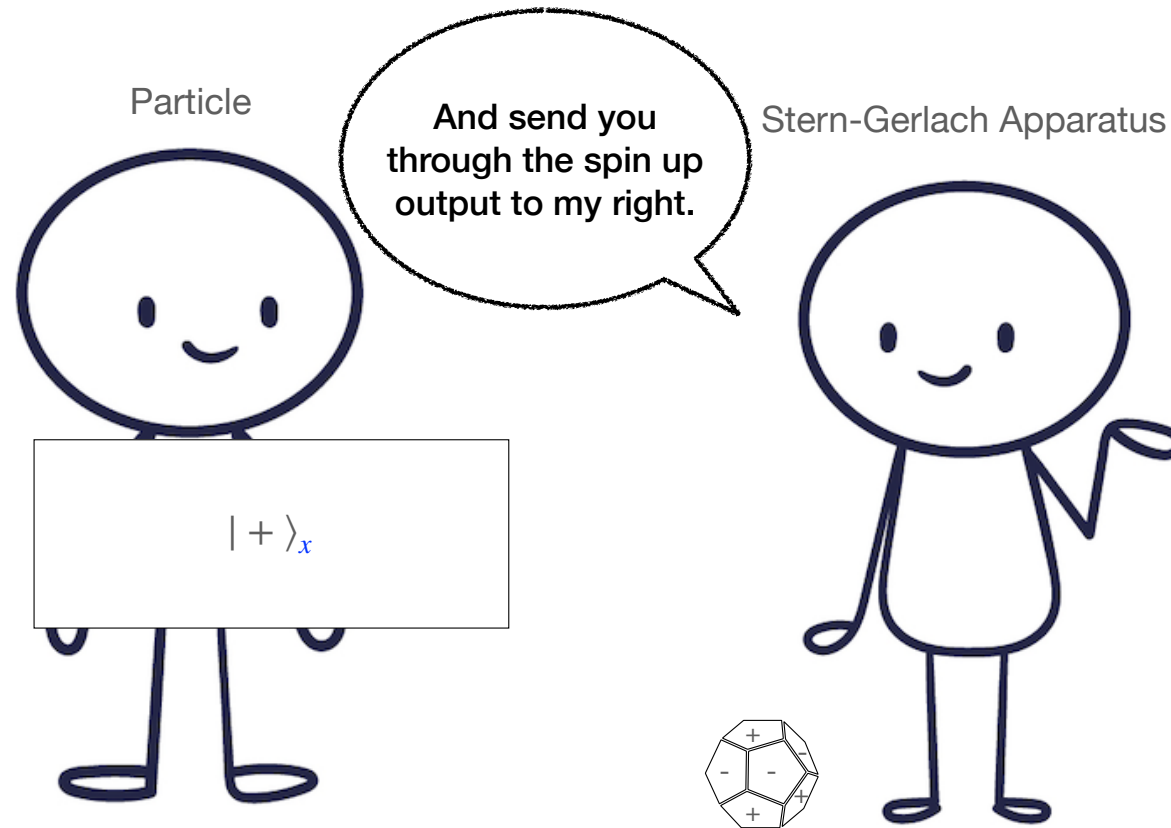
Quantum Measurement Skit



Quantum Measurement Skit



Quantum Measurement Skit



Pedagogical Affordances

- ✓ Emphasizes the probabilistic nature of measurement
- ✓ Probabilities determined by the state
- ✓ *Which* probabilities dictated by the measurement process
- Descriptive rather than explanatory
- Doesn't describe the mechanism of collapse

Future Work

PER about

- ➔ reasoning during inner product activity
- ➔ kinesthetic activities & student identity
- ➔ pedagogical affordances

Frye, MS Project

Hahn Dissertation, Oregon State, 2022

More activities to be developed

Paradigms in Physics

paradigms.oregonstate.edu



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This site is under construction, and currently the easiest way to find curricular material that you're interested in is with the search bar on the upper right.

If you're interested in browsing our content, probably the most useful approach is to browse the [sequences of activities](#) and [homework](#).

Activities 	Sequences 	Homework Problems
Teaching Tips 	Courses 	Learning Progressions

Find Activity Sequences!

Search for Activities! With Instructor Guides

View Whole Courses!



Visit our [OSU PER group website](#) for more information about related research.

Featured Searches:

- quantum
- angular momentum
- spin
- arms
- kinesthetic
- "Raising Physics to the Surface"

Thank You!

This Talk

liz.gire@oregonstate.edu

paradigms.oregonstate.edu