Kinesthetic Activities for Learning Quantum Mechanics

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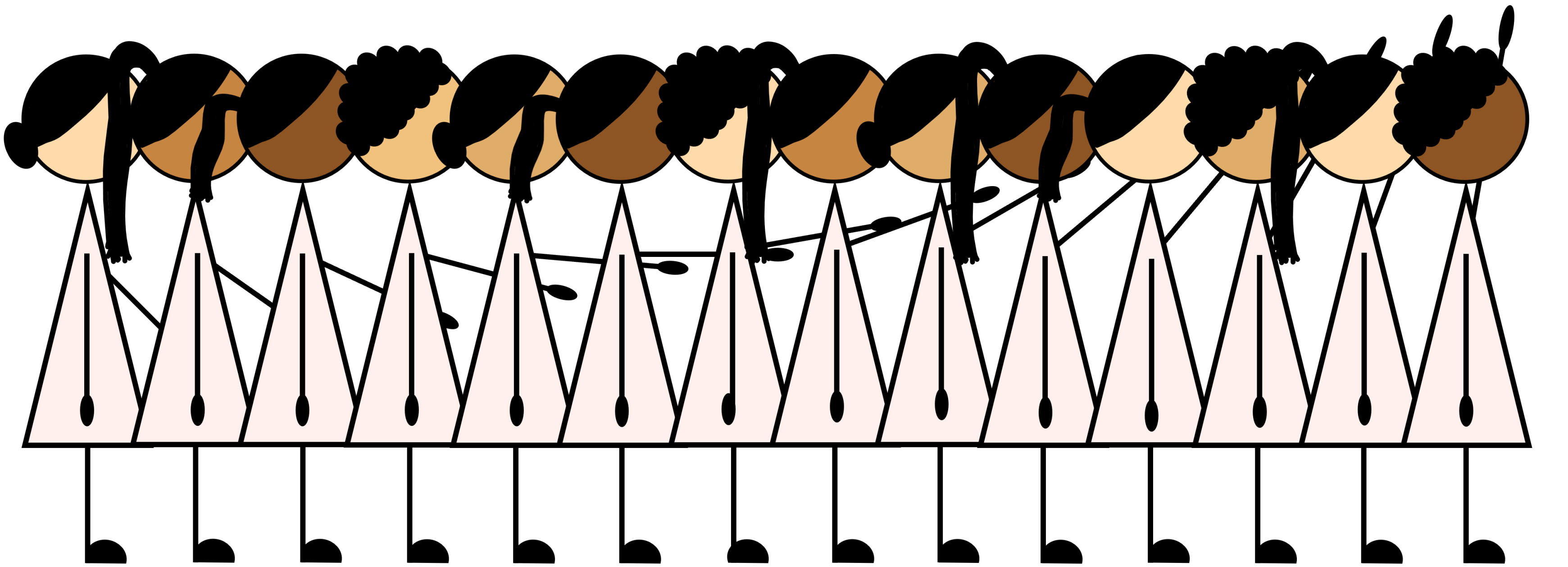
Oregon State University
AAPT Summer 2023



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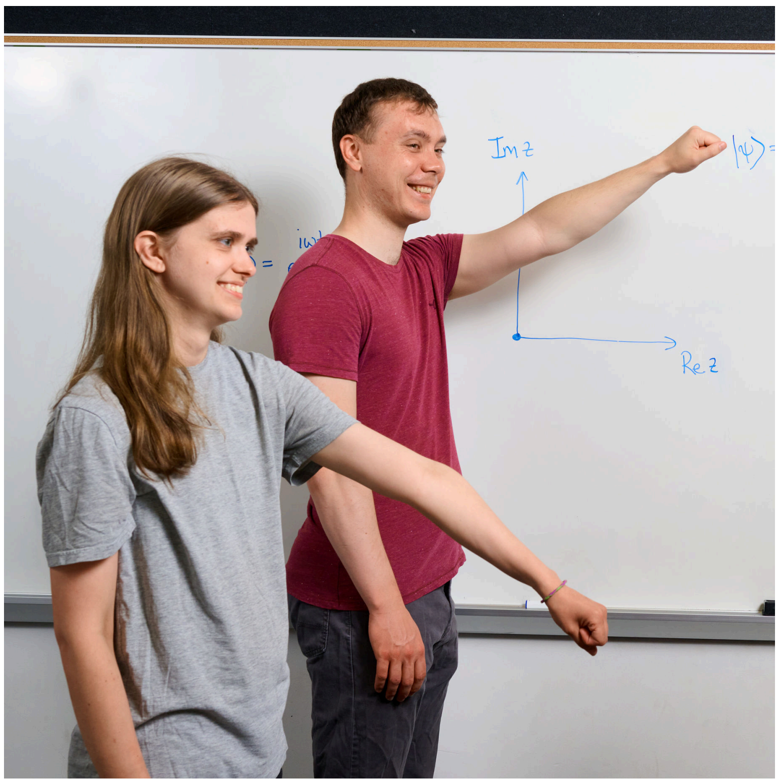
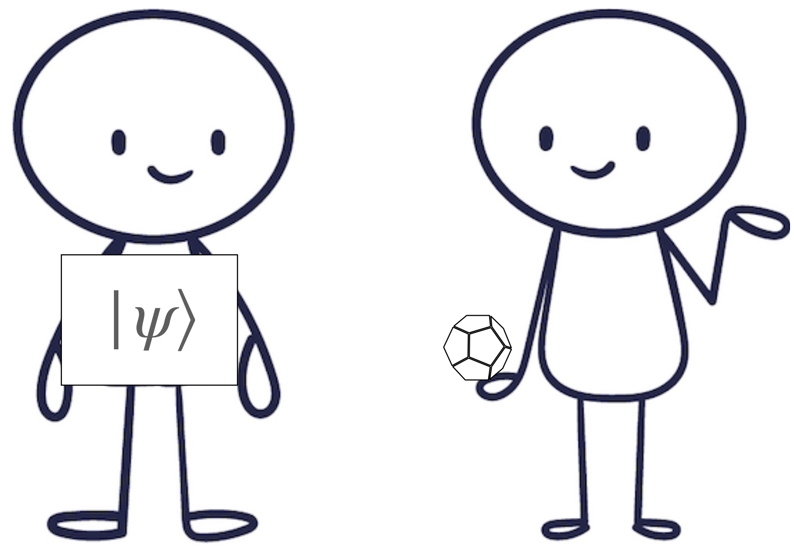
Arms Representation of Quantum States



Quantum Measurement Skit



Kinesthetic Activities for *Upper Division Quantum Mechanics*?!

Activate sensorimotor brain systems

Make decisions about how configure and move sequentially

Re-representation

For quantum systems (>1 people), have to socially negotiate

Introduces silliness and laughter

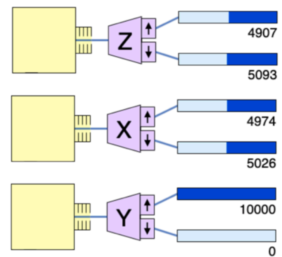
Formative assessment

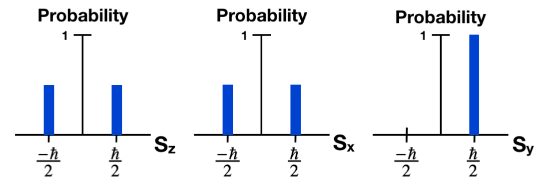
Solomon, et al., *Phys. Ed.*, 1991
Kontra, et al., *Psychol. Sci.*, 2015
Duijzer, et al., Educ. *Psychol. Rev.*, 2019
Struck & Yerrick, *J. Sci Educ. Technol.*, 2010,
Beichner, et al., *Am. J. Phys.*, 1990
Hubber , Titler, & Haslam, *Res. Sci. Educ.*, 2010

Instructional Context

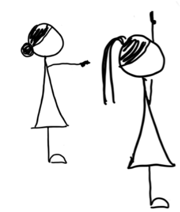
Paradigms in Physics
Quantum Fundamentals & Central Forces Courses

- “Spins First” Approach (McIntyre textbook)
- Stern-Gerlach Simulation to explore postulates of quantum mechanics
- Emphasize Multiple Representations
- Computational lab

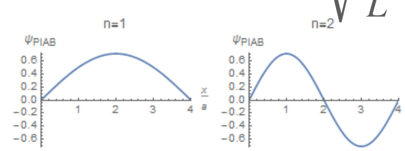


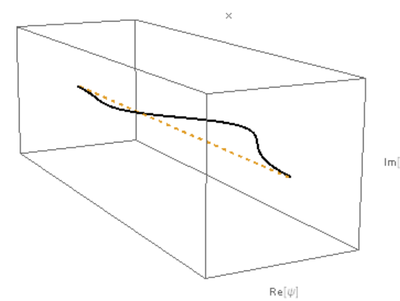


$$\begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix} \qquad 1/\sqrt{2}\,|+\rangle + i/\sqrt{2}\,|-\rangle$$



$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

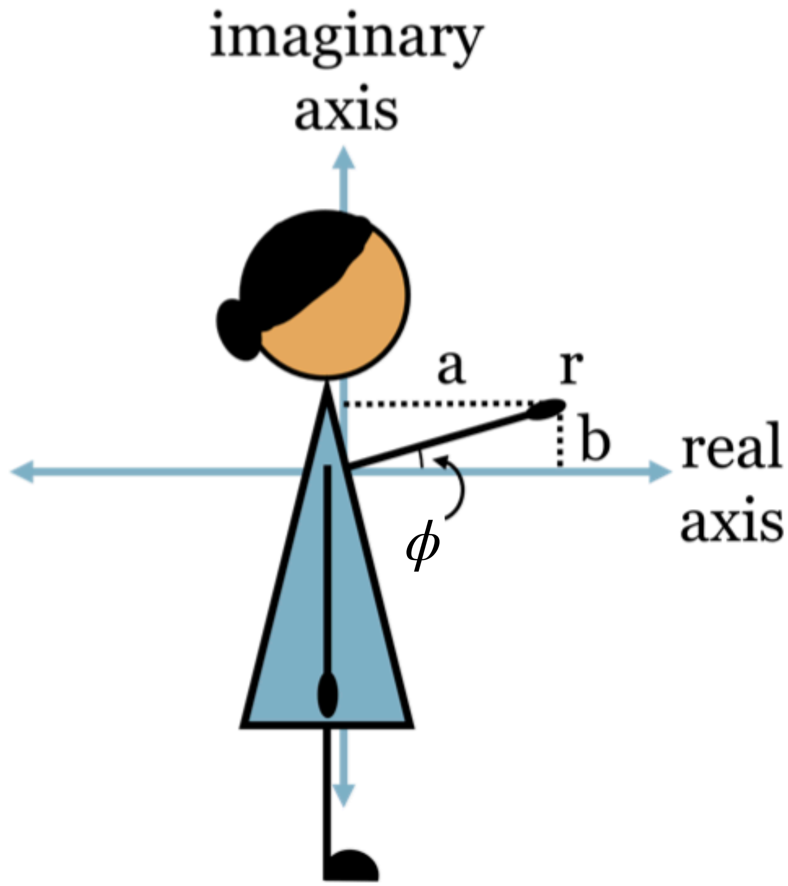




Emigh, et al., *Phys. Rev. PER*, 2020
Manogue, et al., *Am. J. Phys.*, 2001

Arms Representation

Arms Basics



Arms Pros & Cons

- ✓ 4D
- ✓ Phase Angle Salient
- ✓ Accommodate Physical Ability
- ✓ Components of complex numbers vs. quantum basis
- ✓ Memorable

Hahn & Gire, Am. J. Phys., 2022

Arms Pros & Cons

✓ 4D

✓ Phase Angle Salient

✓ Accommodate Physical Ability

✓ Components of complex numbers vs. quantum basis

✓ Memorable

– Arm length not adjustable for different norms

– Lots of information that is not externalized

– Visualization?

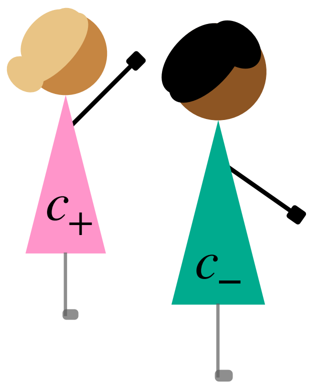
– Self Consciousness

Hahn & Gire, Am. J. Phys., 2022

Quantum Concepts & Representations

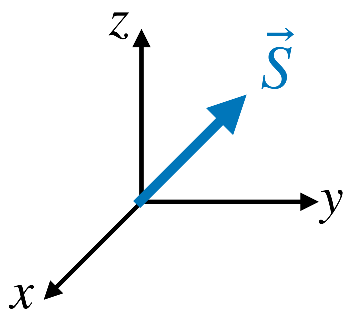
Quantum states are vectors with complex components

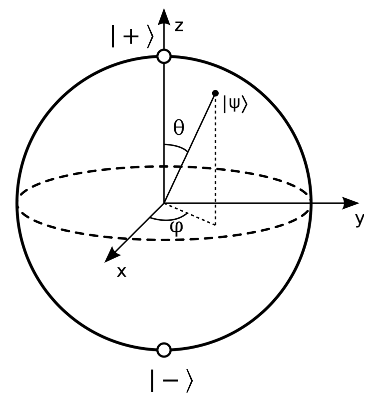
$$|\psi\rangle = c_+|+\rangle + c_-|-\rangle \qquad |\psi\rangle \doteq \begin{bmatrix} c_+ \\ c_- \end{bmatrix}$$

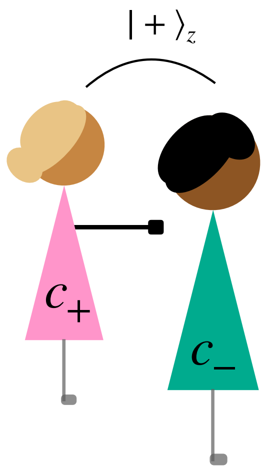


Quantum Concepts & Representations

Cartesian space and Hilbert space are different





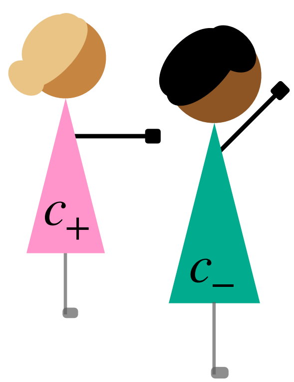


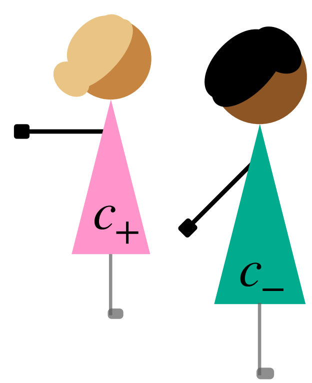
$$|\psi\rangle = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}e^{i\varphi}|-\rangle$$

Quantum Concepts & Representations

Vectors that differ by an overall phase represent the same quantum state

$$|\psi\rangle = c_+ |+\rangle + c_- |-\rangle \qquad\qquad |\psi\rangle = e^{i\phi}\Big(c_+ |+\rangle + c_- |-\rangle\Big)$$

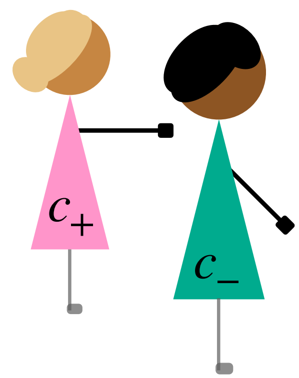




Quantum Concepts & Representations

Quantum states evolve with time - time & energy-dependent phase on terms in energy eigenstate expansion

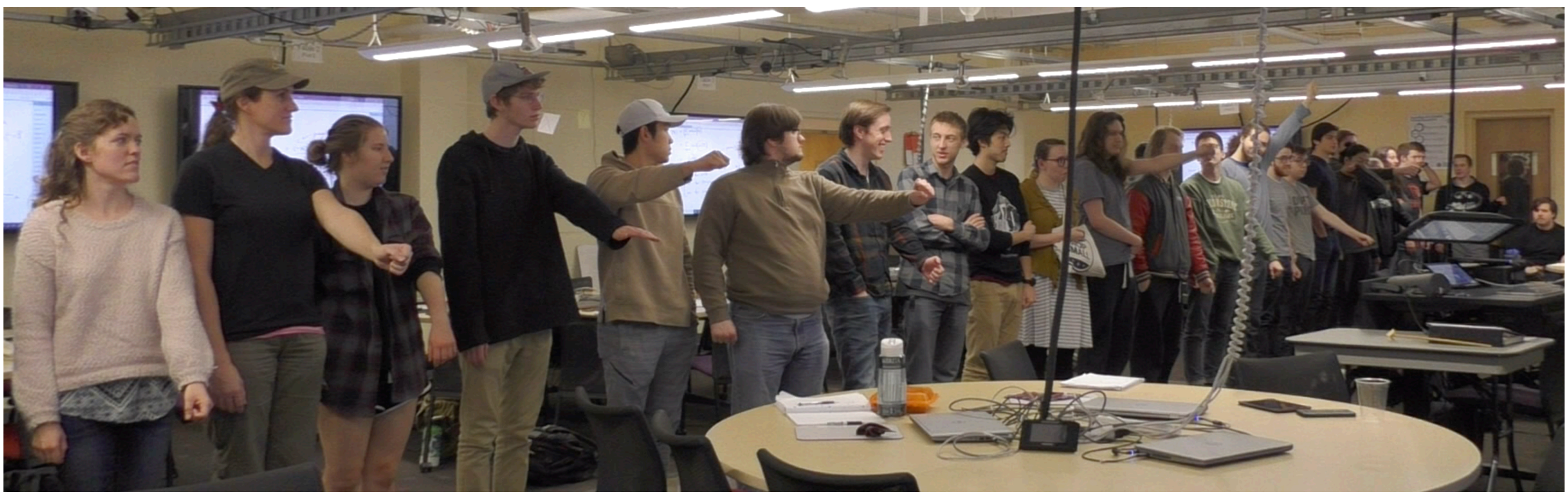
$$|\psi(t)\rangle = c_+ e^{-iE_+t/\hbar}|+\rangle \; + \; c_- \; e^{-iE_-t/\hbar}|-\rangle$$



Quantum Concepts & Representations

Formalisms for discrete and continuous quantum systems are related.

$$c_\pm = {}_z\langle \pm | \psi \rangle \qquad \psi(x) = \langle x | \psi \rangle$$



Arms Activities

Complex Numbers Quantum State Relative & Overall Phase Time Evolution Wavefunction	Hahn & Gire, *Am. J. Phys*, 2022
Inner Product of Spin-1/2 States Time Evolution of a Particle on a Ring	This talk

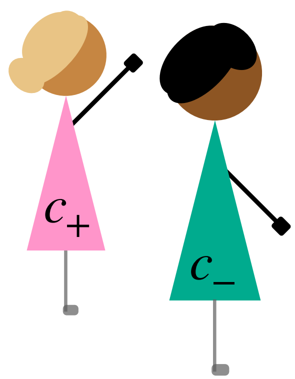
Inner Product of Spin-1/2 System

Quantum Concepts & Representations

Measurement probabilities are related to inner products between quantum states

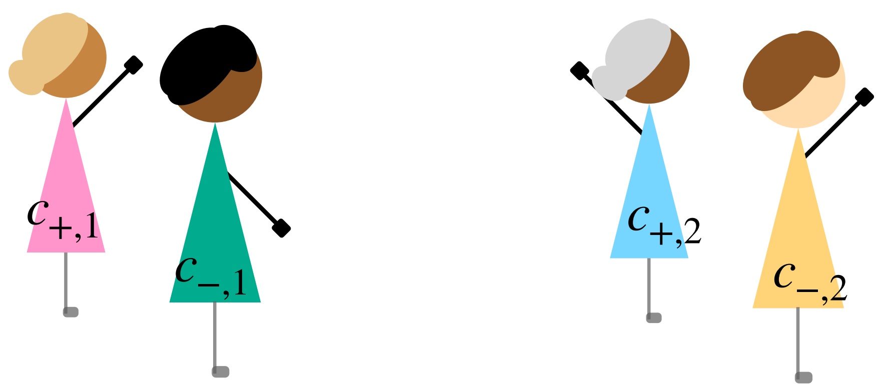
$$\mathscr{P}\left(S_z = \frac{+\hbar}{2}\right) = \left|\ {}_z\langle + | \psi \rangle\right|^2$$

Inner Product of Spin-1/2 System



Ask pair of students to represent
an arbitrary state.

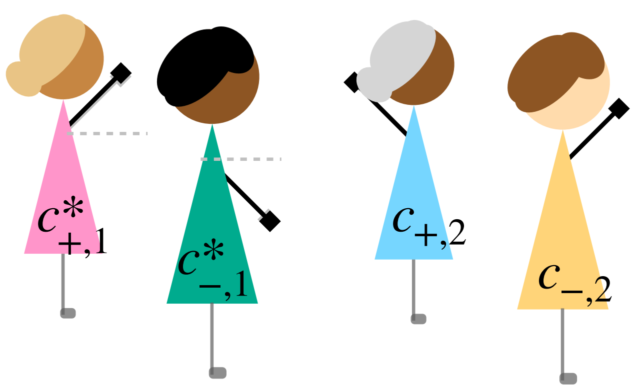
Inner Product of Spin-1/2 System



Introduce a second state (each rotated by π/2)

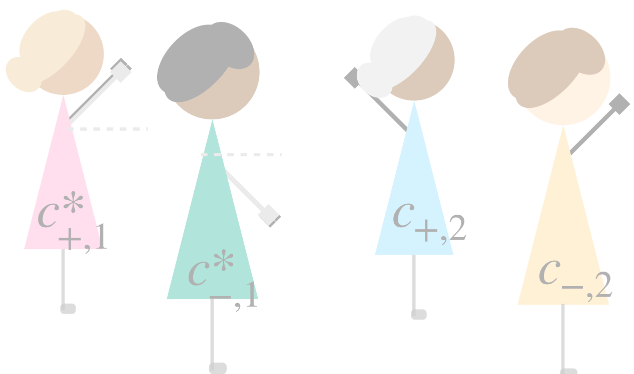
Are these states orthogonal?

Inner Product of Spin-1/2 System

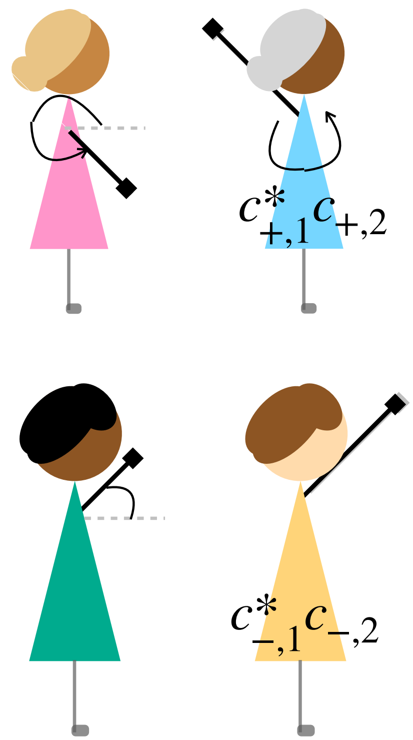


Complex Conjugate 1 pair

Inner Product of Spin-1/2 System

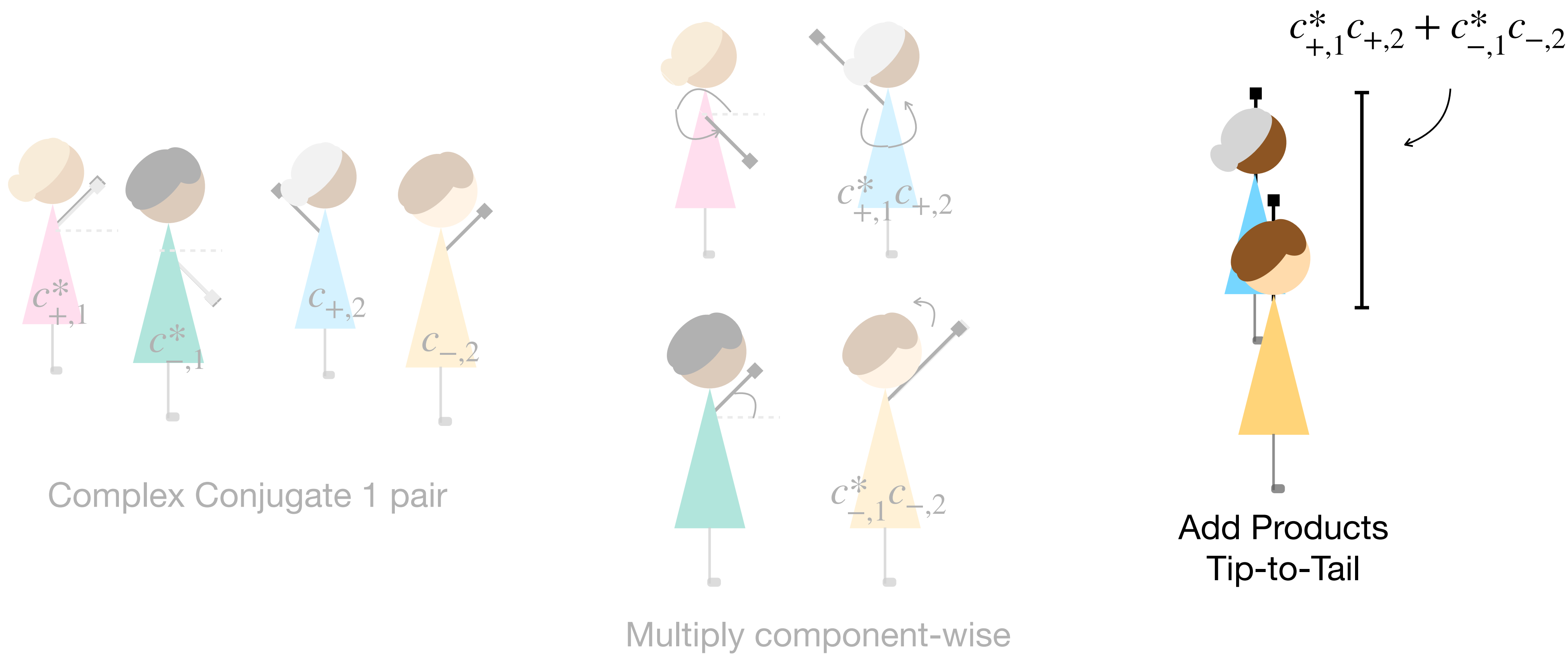


Complex Conjugate 1 pair



Multiply component-wise

Inner Product of Spin-1/2 System



Pedagogical Affordances

✓ Emphasizes steps, particularly

- complex conjugate
- aligning components

– Arm length not adjustable for different norms

– Adding “tip-to-tail” requires effort

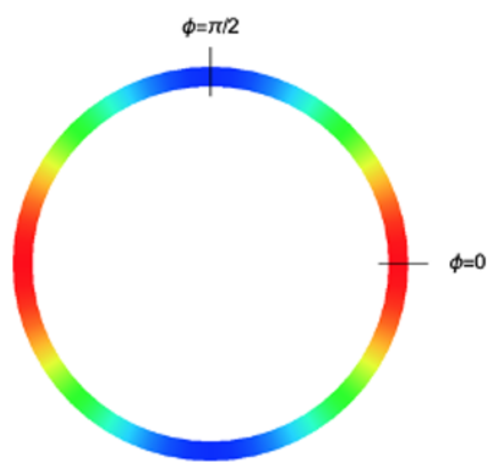
Time Evolution of a Quantum Particle on a Ring

Time Evolution of Particle on a Ring

$$E_m(\phi) \doteq \langle \phi | m \rangle = \frac{1}{\sqrt{2\pi}} e^{im\phi}$$

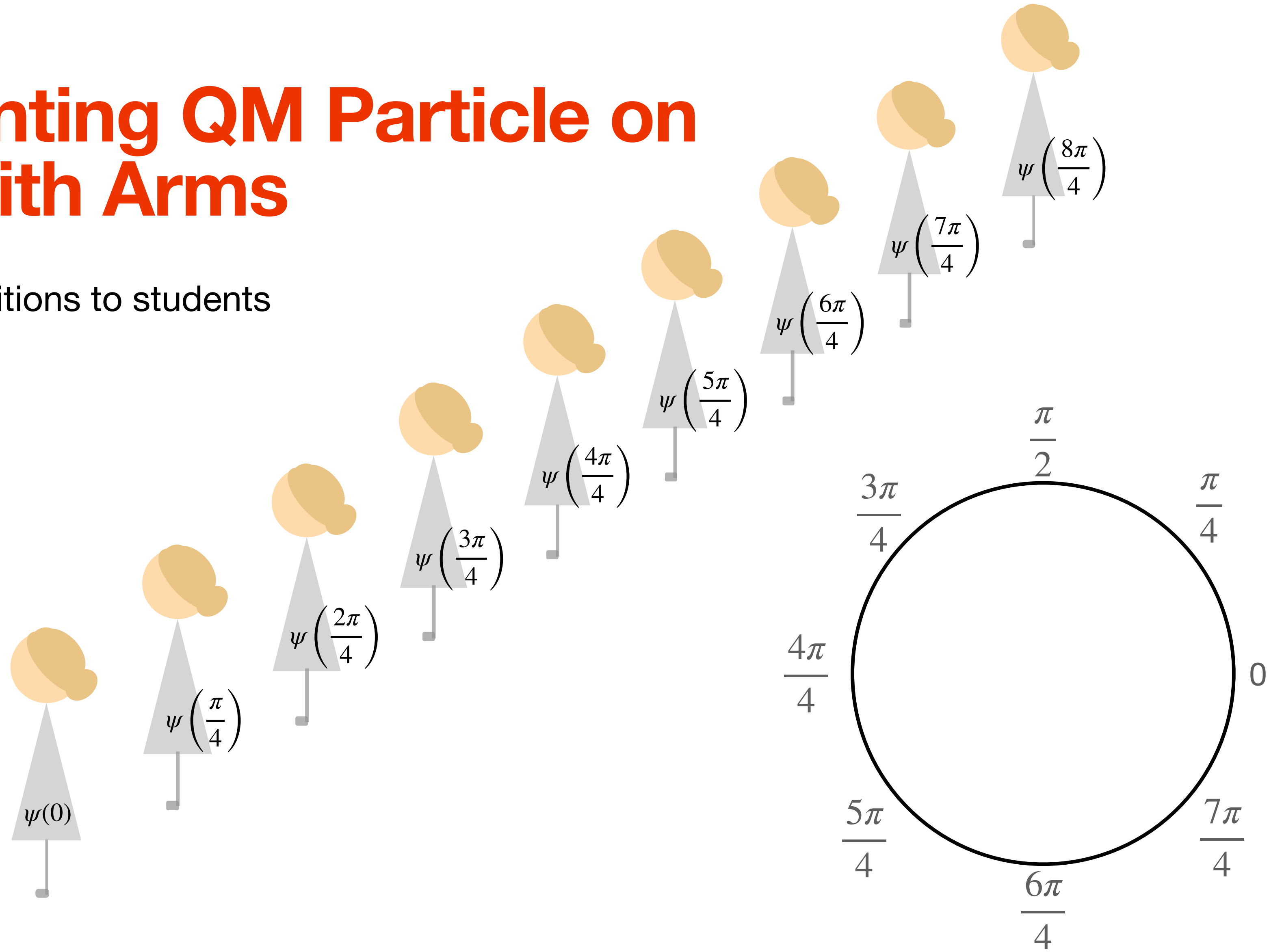
$$E_m = \frac{m^2\hbar^2}{2I}$$

Probability Density for m=1



Representing QM Particle on a Ring with Arms

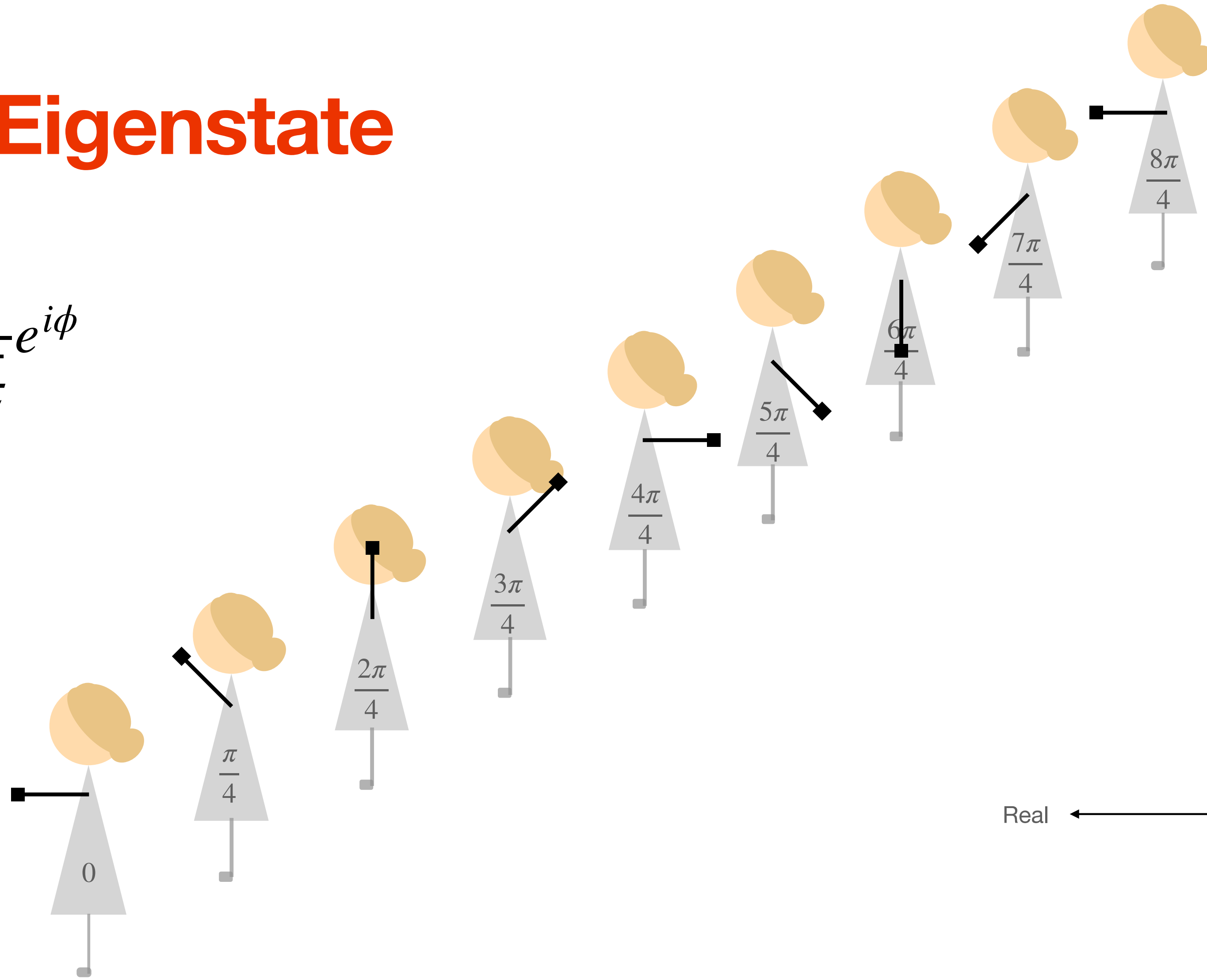
Assign angular positions to students

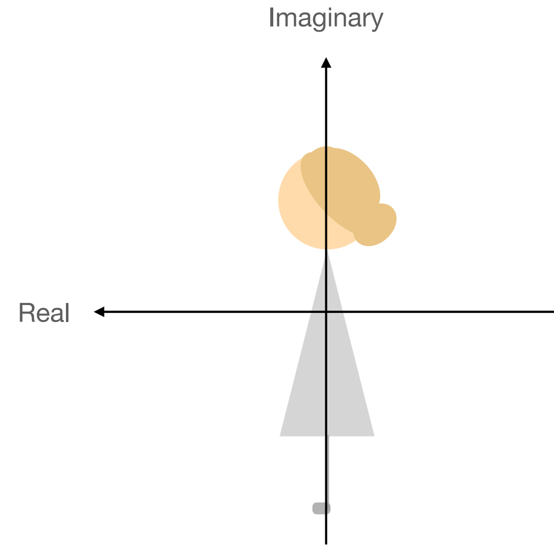


Energy Eigenstate

m=1

$$E_1(\phi) = \frac{1}{\sqrt{2\pi}} e^{i\phi}$$



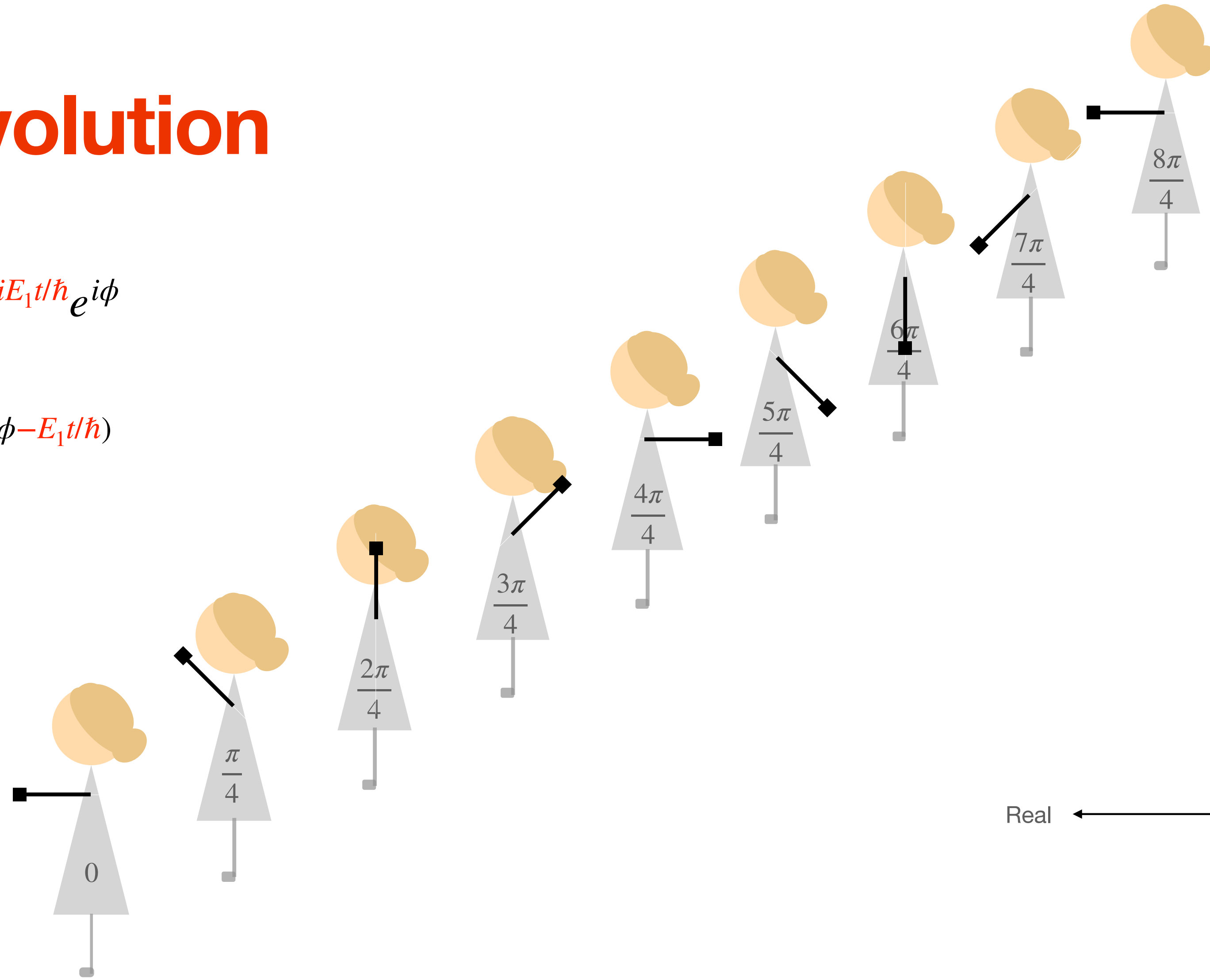


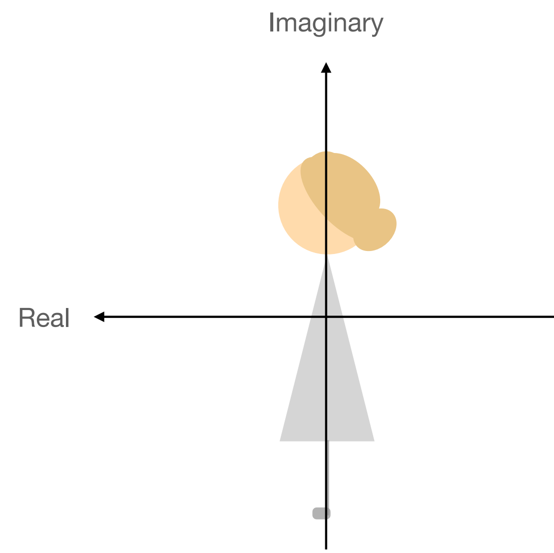
Time Evolution

m=1

$$E_1(\phi) = \frac{1}{\sqrt{2\pi}} e^{-iE_1 t/\hbar} e^{i\phi}$$

$$= \frac{1}{\sqrt{2\pi}} e^{i(\phi - E_1 t/\hbar)}$$



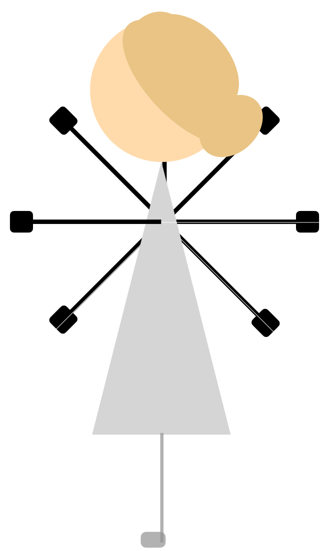


Time Evolution - Shoulder View

m=1

$$E_1(\phi) = \frac{1}{\sqrt{2\pi}} e^{-iE_1 t/\hbar} e^{i\phi}$$

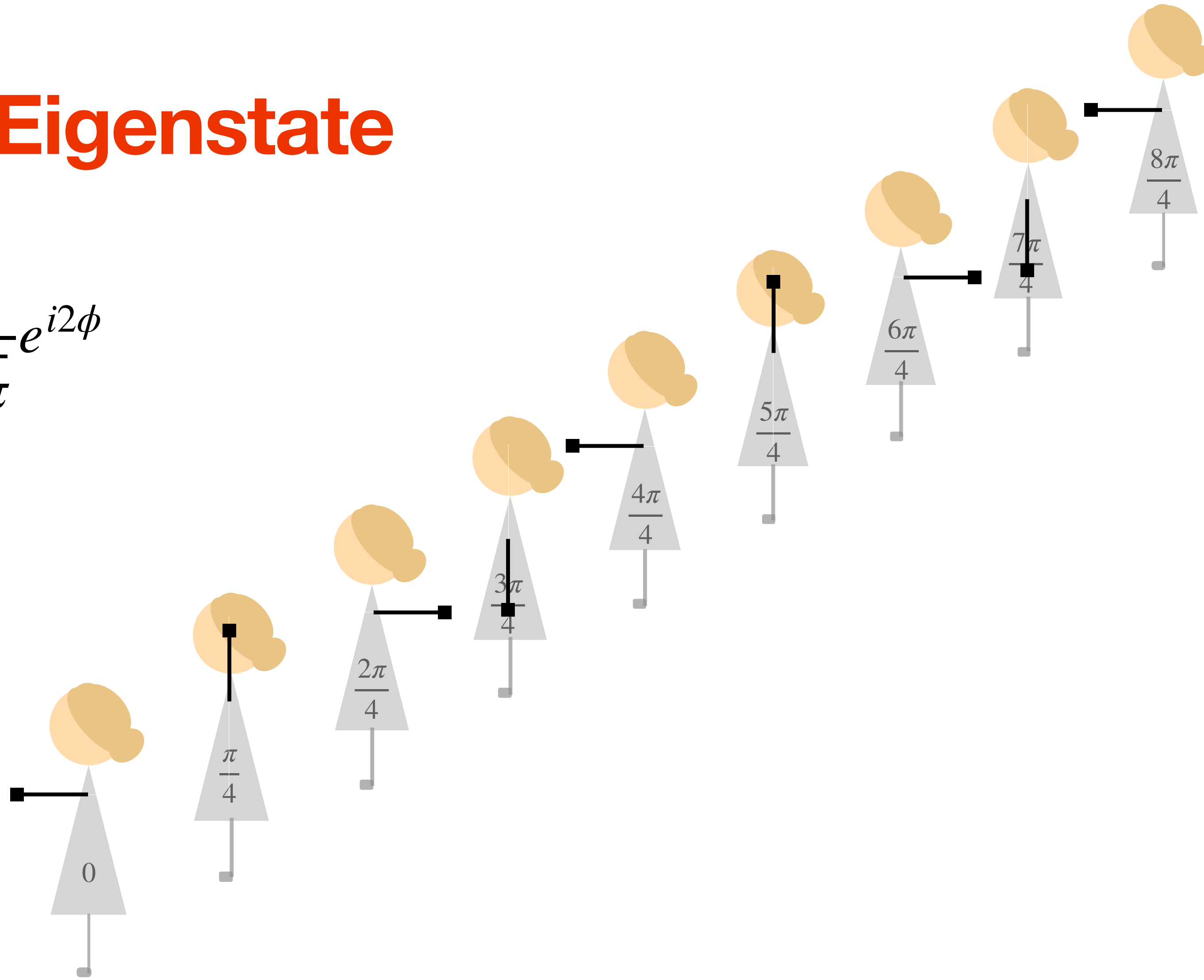
$$= \frac{1}{\sqrt{2\pi}} e^{i(\phi - E_1 t/\hbar)}$$



Energy Eigenstate

m=2

$$E_2(\phi) = \frac{1}{\sqrt{2\pi}} e^{i2\phi}$$



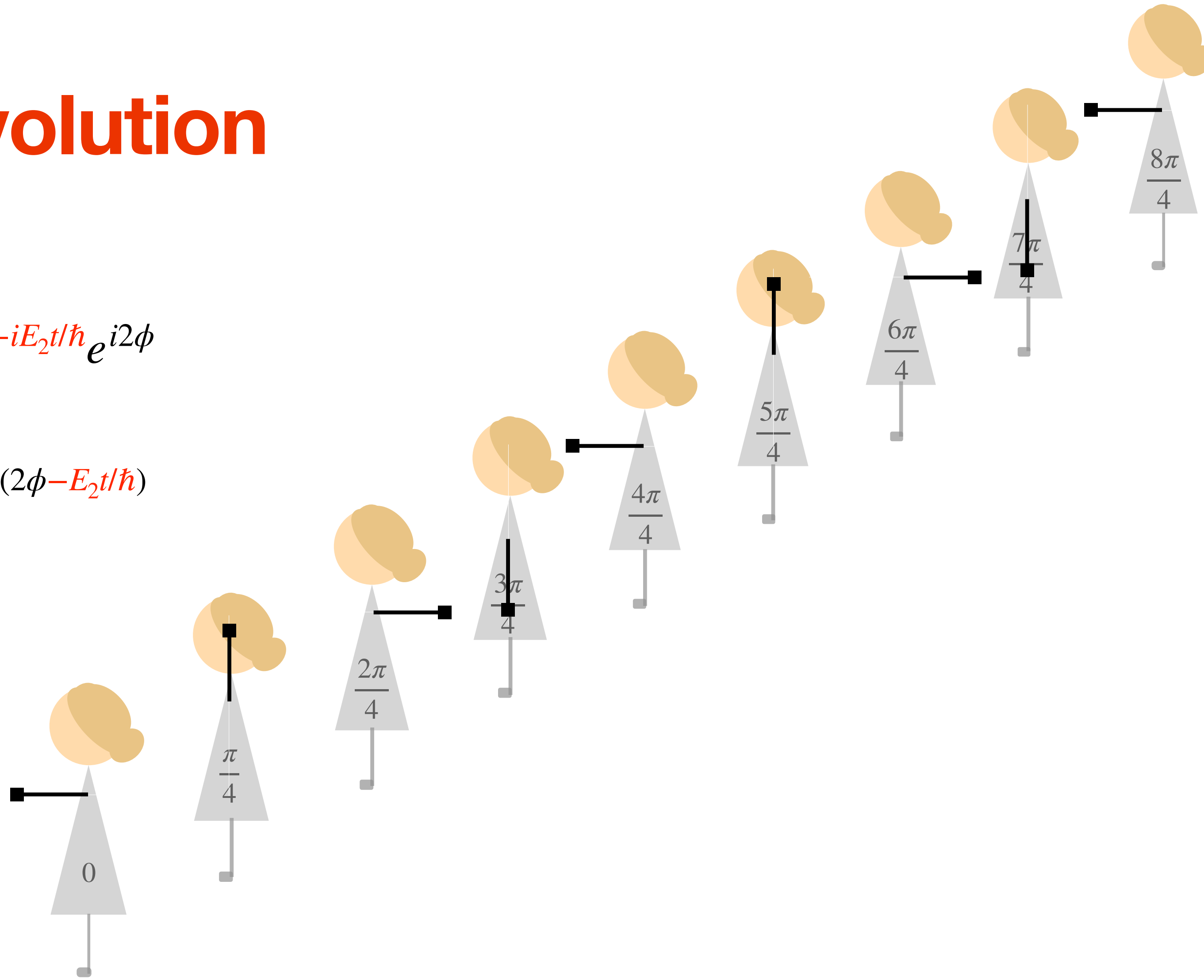
Time Evolution

m=2

$$E_2(\phi) = \frac{1}{\sqrt{2\pi}} e^{-iE_2t/\hbar} e^{i2\phi}$$

$$= \frac{1}{\sqrt{2\pi}} e^{i(2\phi - E_2t/\hbar)}$$

$$E_2 = 4E_1$$



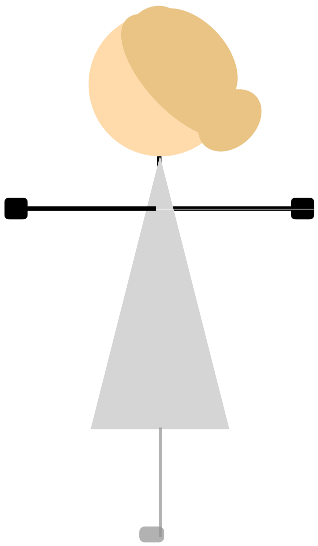
Time Evolution

m=2

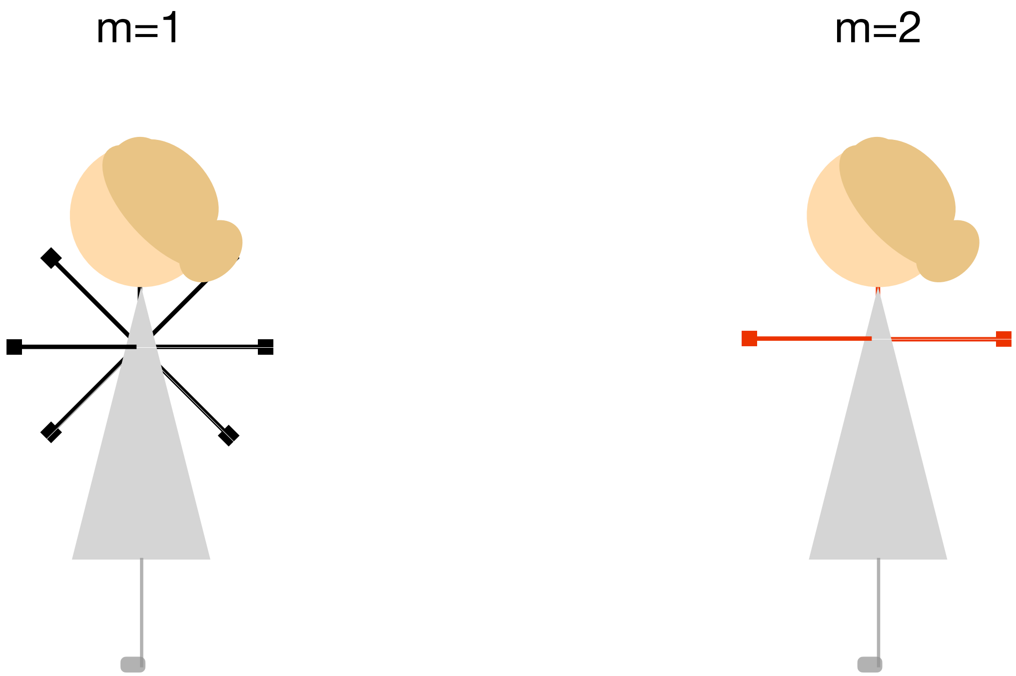
$$E_2(\phi) = \frac{1}{\sqrt{2\pi}} e^{-iE_2 t/\hbar} e^{i2\phi}$$

$$= \frac{1}{\sqrt{2\pi}} e^{i(2\phi - E_2 t/\hbar)}$$

$$E_2 = 4E_1$$

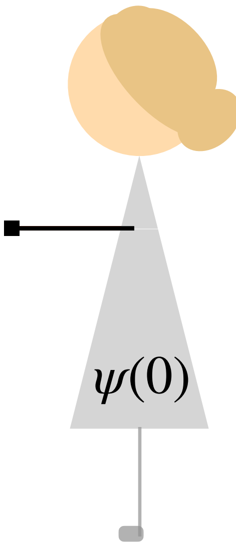


Time Evolution



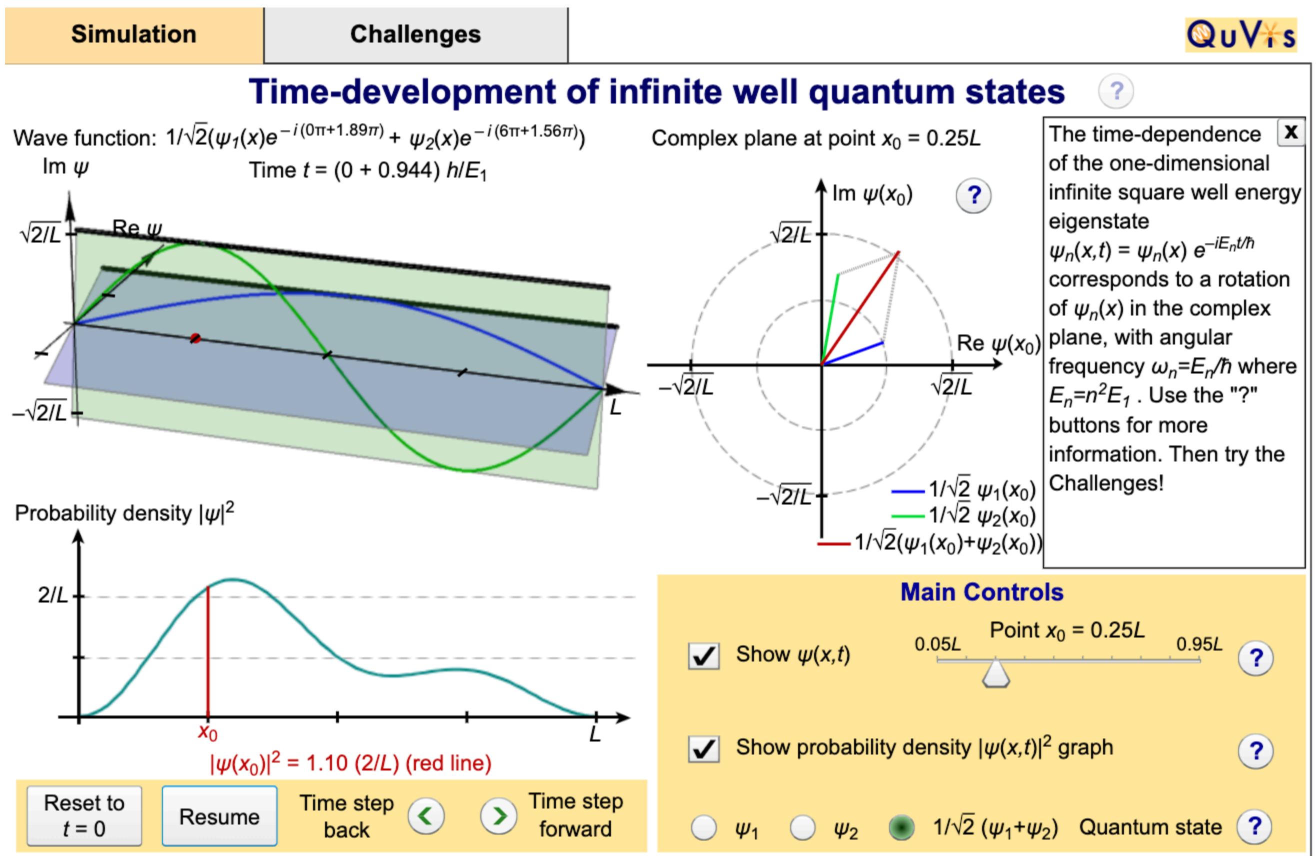
$$E_2 = 4E_1$$

Superposition



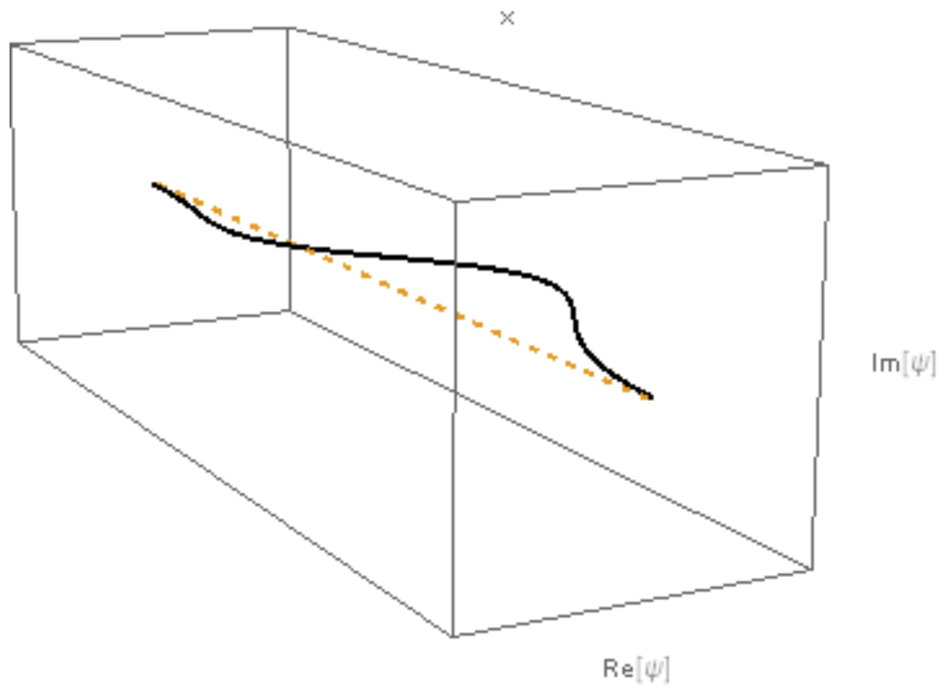
QuVis (St Andrews)

https://www.st-andrews.ac.uk/physics/quvis/



Graphical Superposition

Infinite Square Well



Pedagogical Affordances

- ✓ For eigenstates, arms are norm=1
- ✓ Highlights differences between stationary and non-stationary states
- ✓ Superposition at each position results in complicated time evolution

- – Requires at least 8 students
- – Completing the superposition is difficult

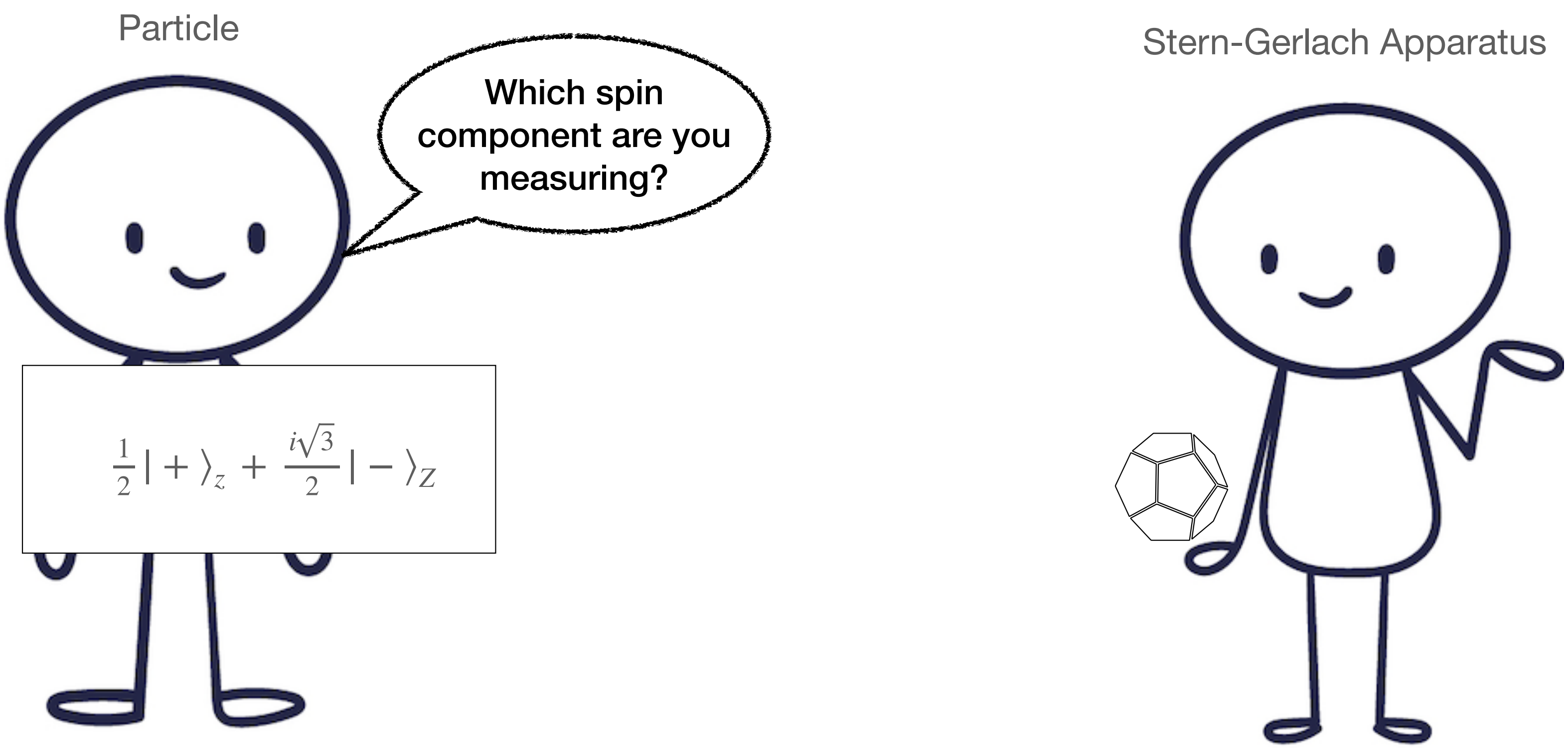
Quantum Measurement Skit

Quantum Concepts & Representations

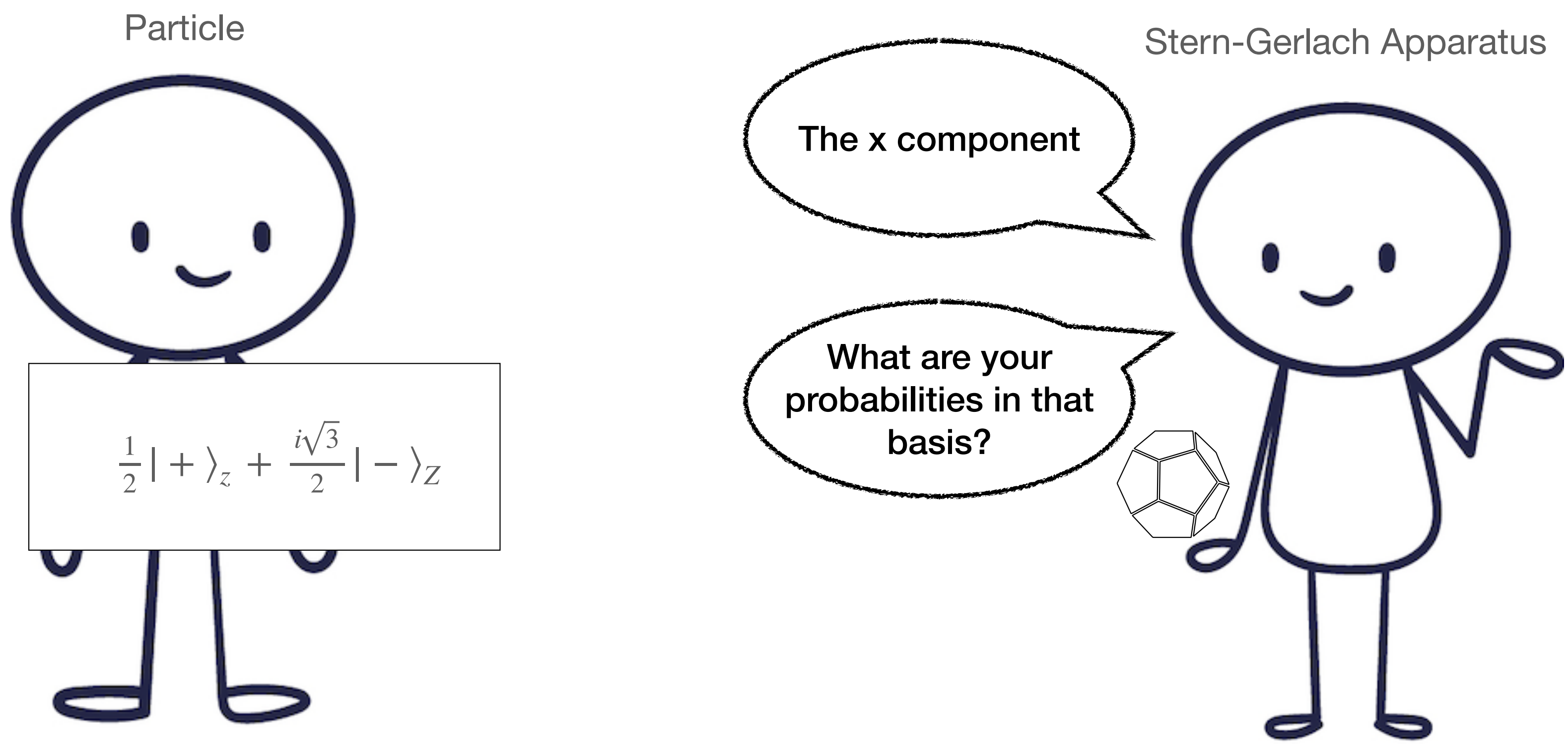
Measurement results in a probabilistic projection onto the output basis and renormalization

$$|\psi_{out}\rangle = \frac{\hat{P}|\psi_{in}\rangle}{\langle\psi_{in}|\hat{P}|\psi_{in}\rangle}$$

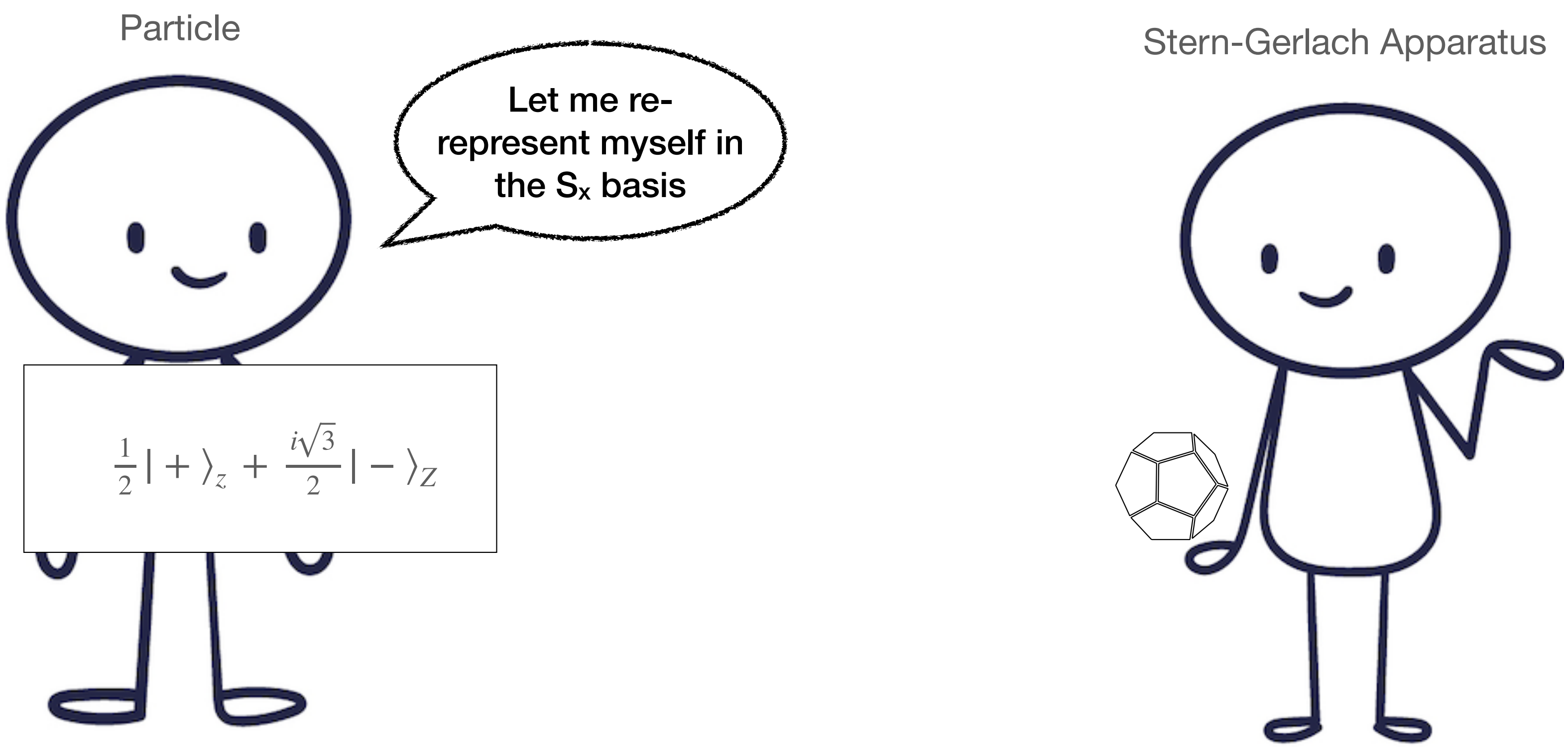
Quantum Measurement Skit



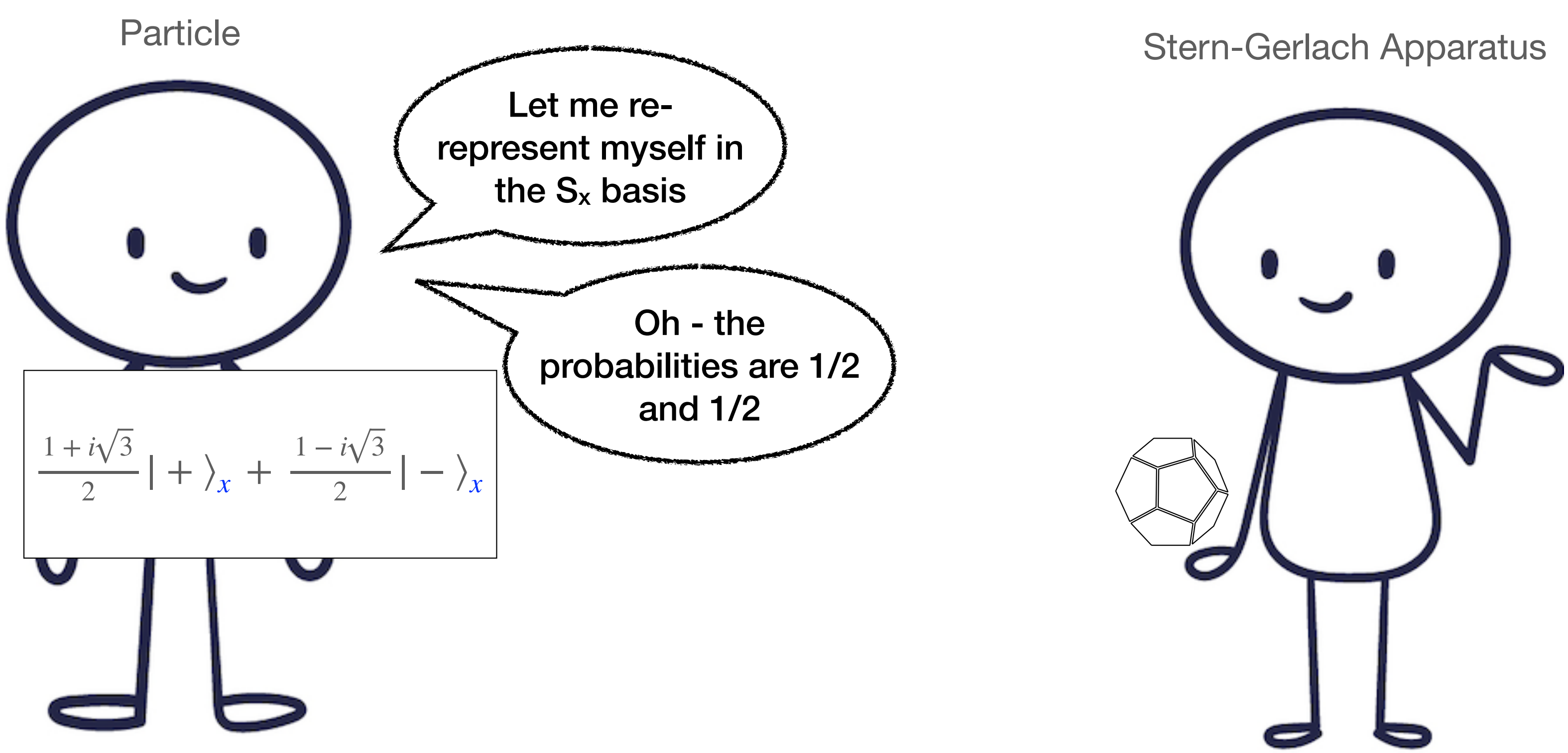
Quantum Measurement Skit



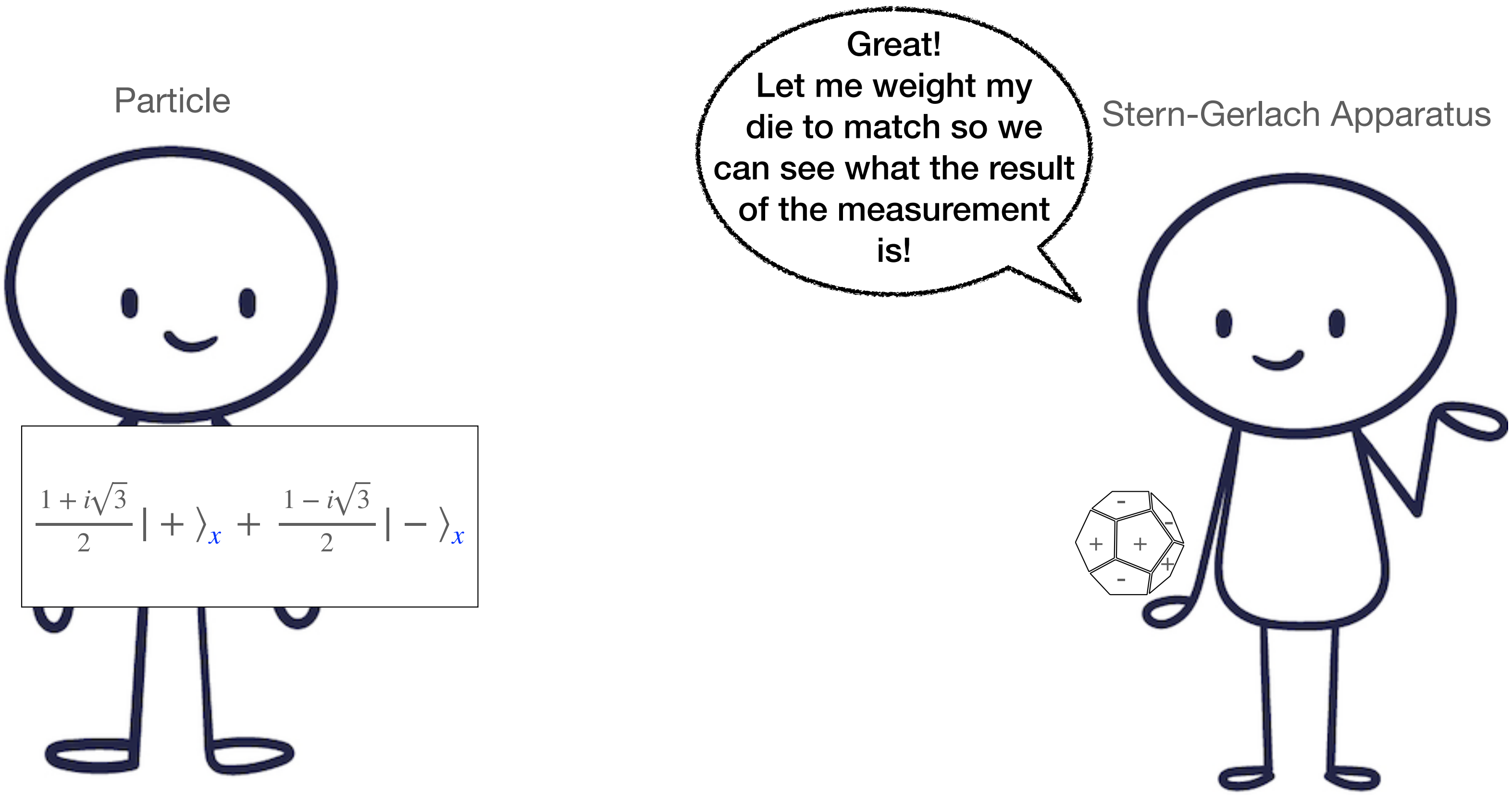
Quantum Measurement Skit



Quantum Measurement Skit

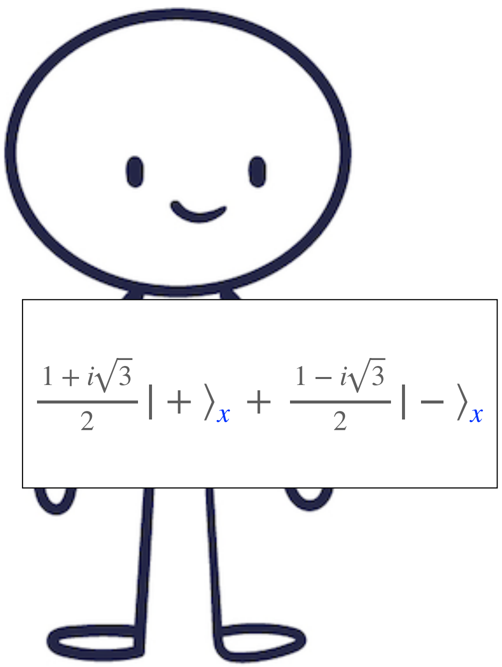


Quantum Measurement Skit

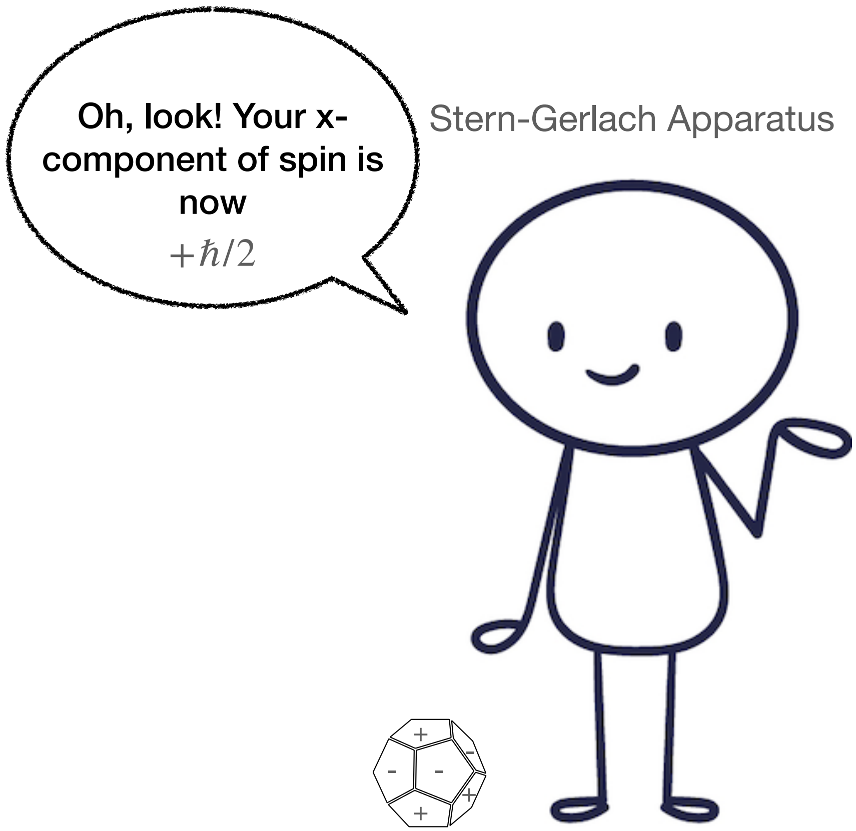


Quantum Measurement Skit

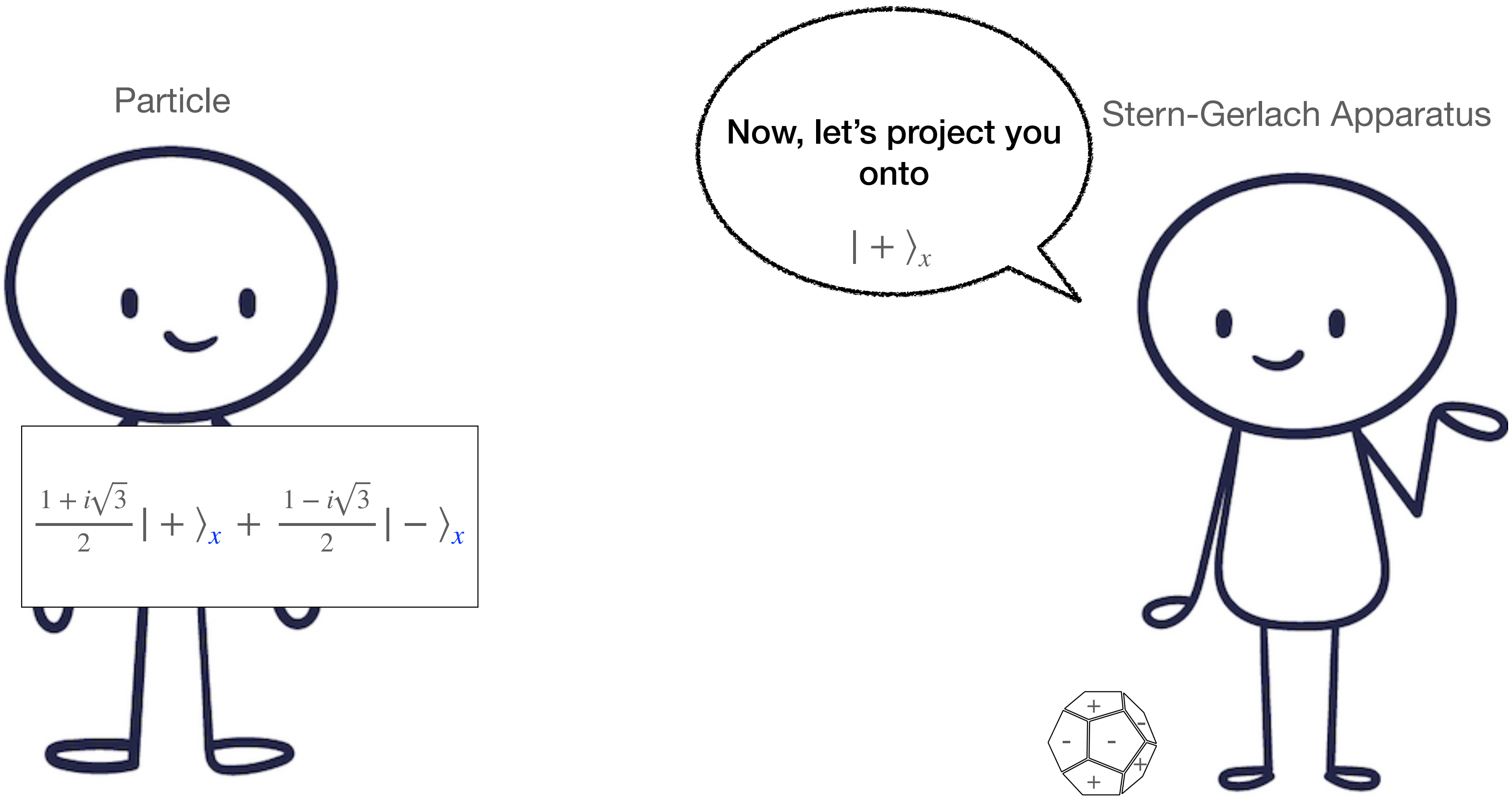
Particle



Stern-Gerlach Apparatus



Quantum Measurement Skit



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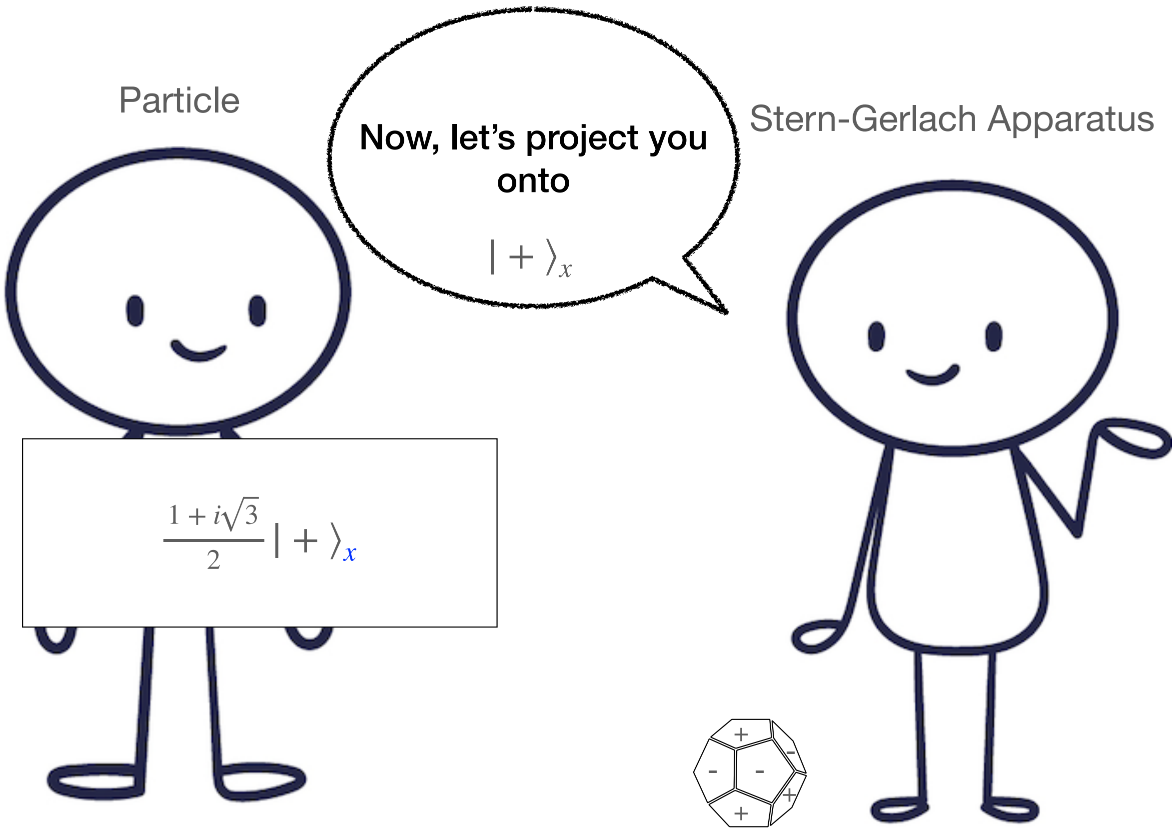

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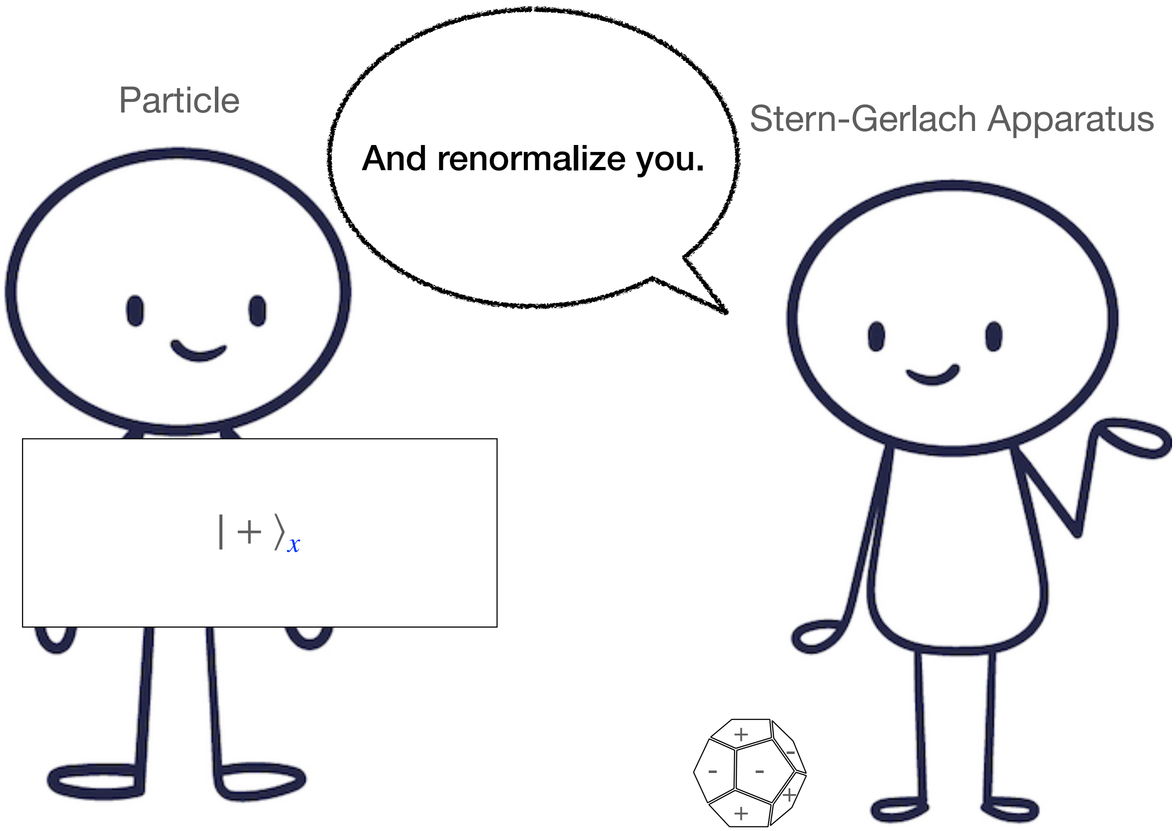

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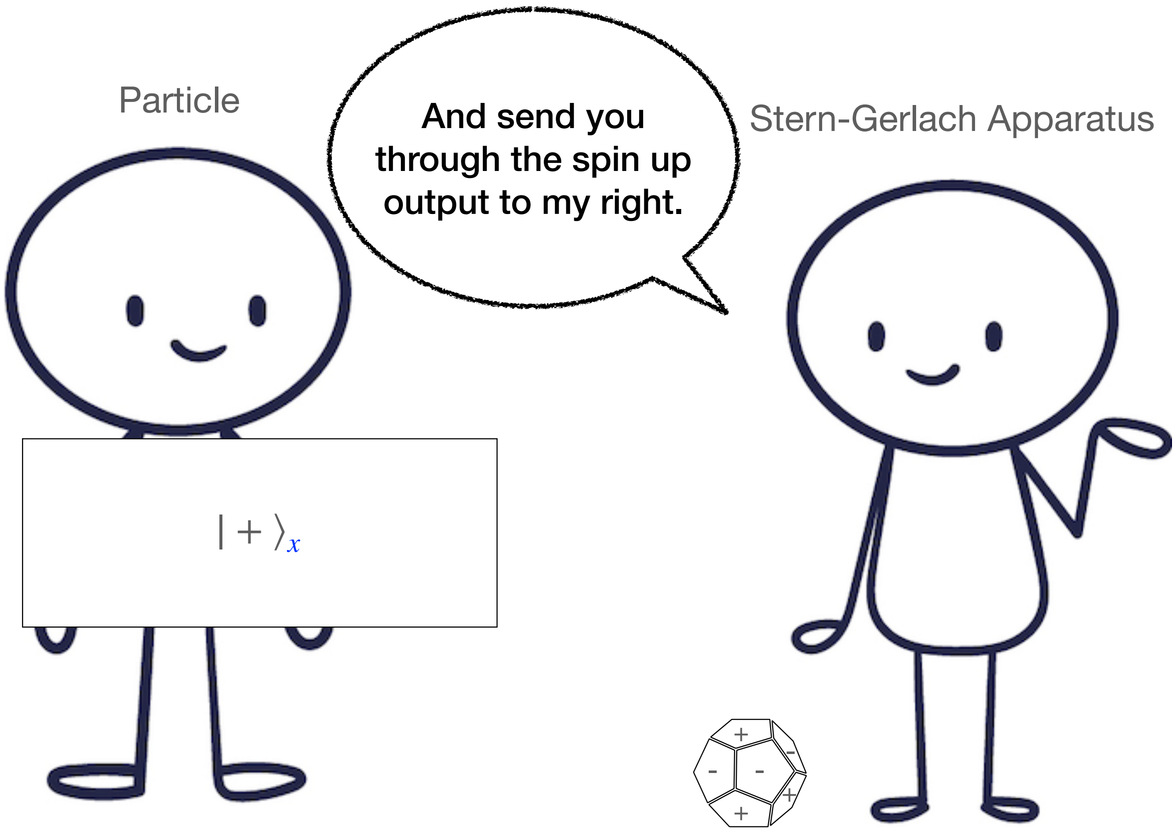


Pedagogical Affordances

- ✓ Emphasizes the probabilistic nature of measurement
- ✓ Probabilities determined by the state
- ✓ *Which* probabilities dictated by the measurement process

- – Descriptive rather than explanatory
- – Doesn't describe the mechanism of collapse

Future Work

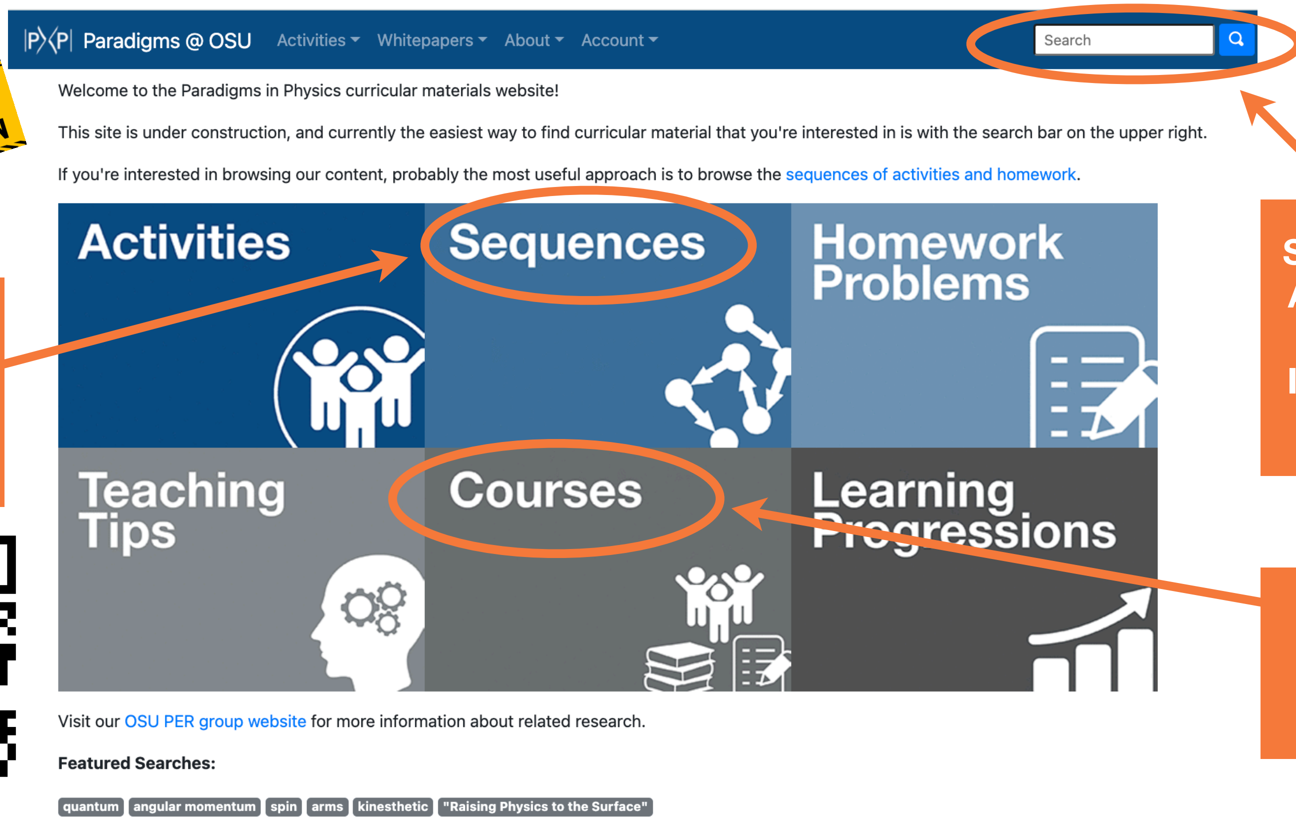
PER about

- ➡ reasoning during inner product activity — Frye, MS Project
- ➡ kinesthetic activities & student identity — Hahn Dissertation, Oregon State, 2022
- ➡ pedagogical affordances

More activities to be developed

Paradigms in Physics

paradigms.oregonstate.edu



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View Whole Courses!

Thank You!

This Talk

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