

# How Students Compare and Contrast the “Discreteness” of Quantum Representations

Christian D. Solorio (he/him)

Elizabeth Gire

David Roundy

Oregon State University

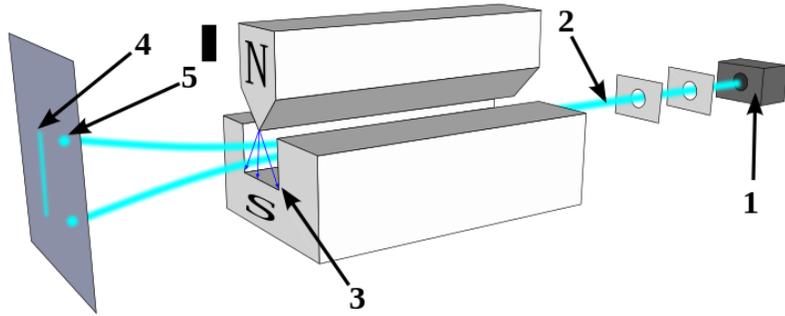


DUE 1836604

DUE 1836604



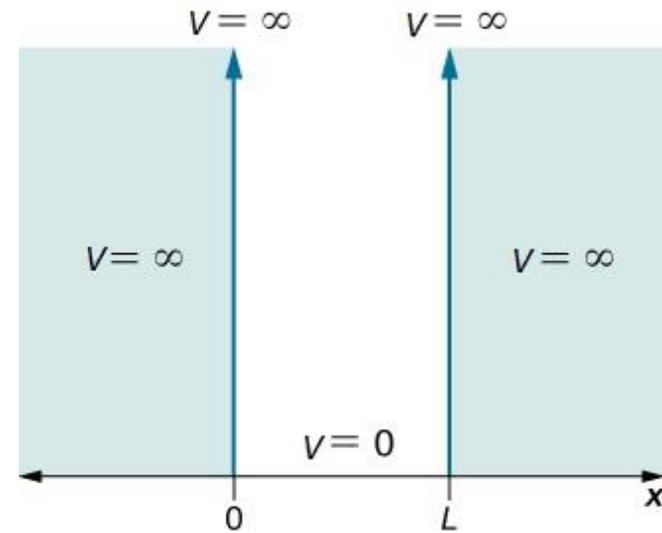
## Spin- $\frac{1}{2}$ Systems



- Discrete, finite bases



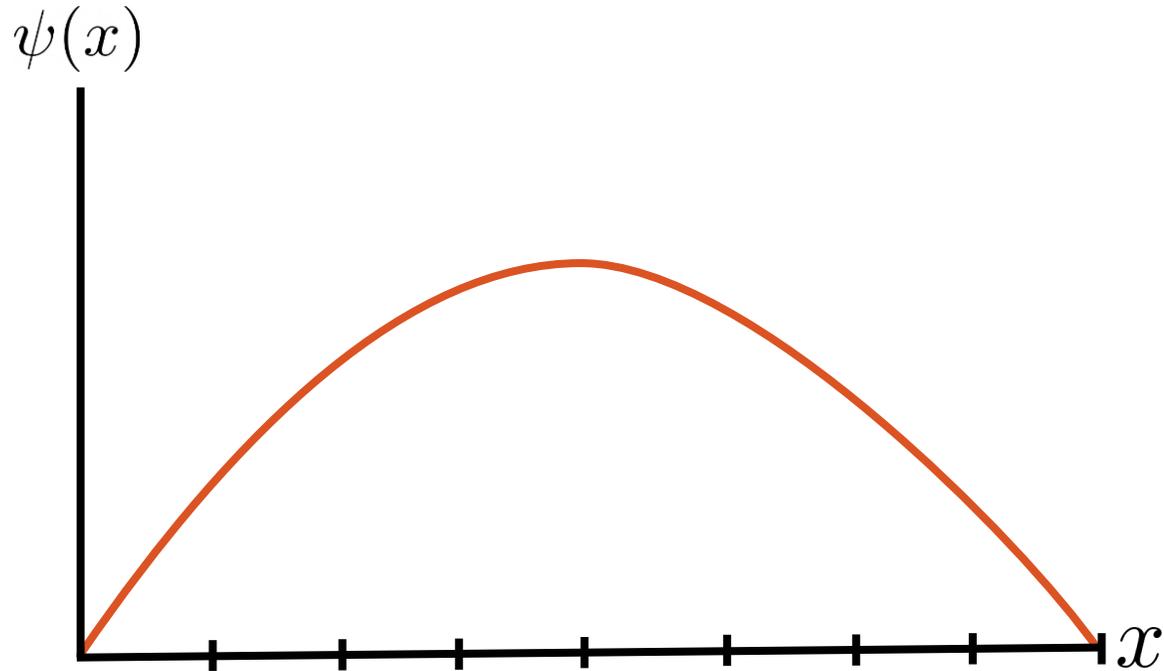
## Infinite Square Well



- Continuous bases

# Discretization!

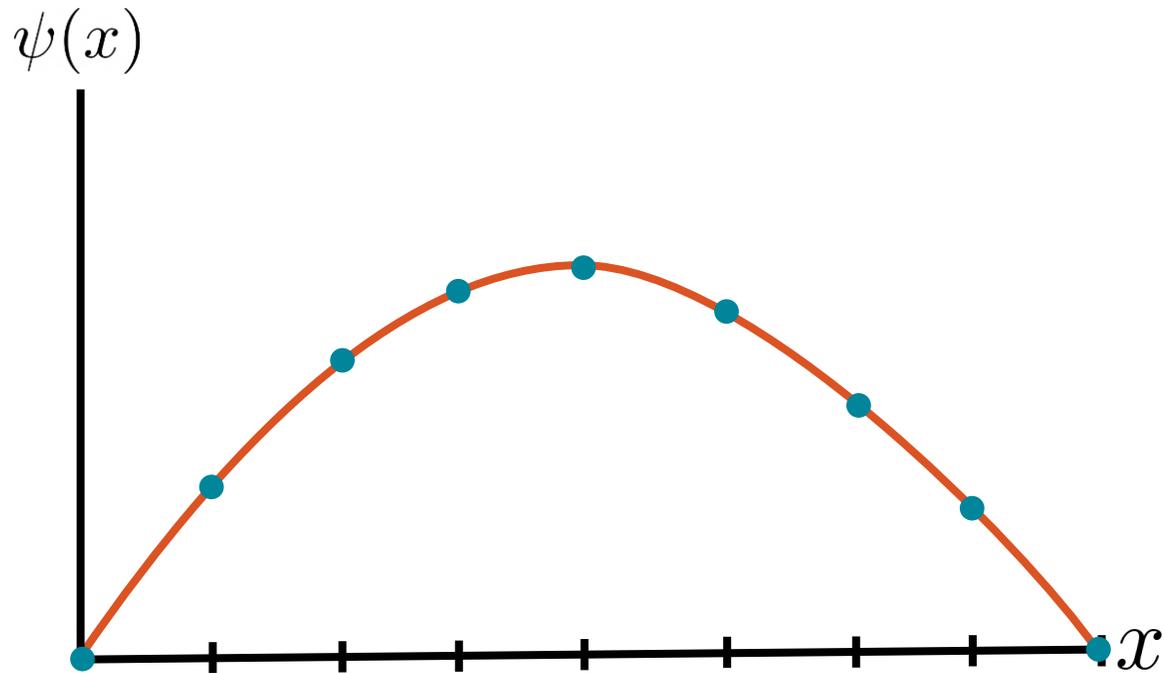
- Wavefunctions are discretized for computational operations



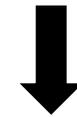
$$\psi(x) = Ax(L - x)$$

# Discretization!

- Wavefunctions are discretized for computational operations



$$\psi(x) = Ax(L - x)$$



$$\psi(x) \rightarrow \{\psi(0), \psi(\Delta x), \psi(2\Delta x), \dots, \psi(L)\}$$

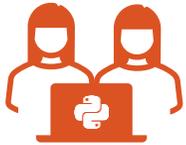
## Research Question

How do students understand  
discreteness in quantum  
mechanics?

# Computational Lab Course Structure



Supports physics learning in the context of programming



Pair-programming in Python



Taught by an instructor and a team of TAs



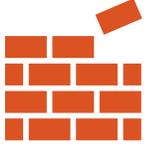
Wavefunctions focused approach



# Card Sorting Task Interview Methods



Conducted semi-structured interviews with 6 participants



Organized 20 quantum mechanics cards



Used Optimal Workshop's OptimalSort



Participants followed a *think aloud* protocol

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$ +\rangle$
$\int \varphi_n^*(x)\psi(x)dx$	$ E_n\rangle$
$ \psi\rangle$	$(1 \ 0) \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$
$ \psi(x) ^2$	<code>dx = 0.01</code>
$\Delta x$	$\varphi_n(x)$
$\langle x \psi\rangle$	$\langle + \psi\rangle$
$ \langle + \psi\rangle ^2$	<pre>L = 1 n = 1 sum = 0 for x in np.arange(0, L, dx):     sum += np.cos(phi(n, x))*Psi(x)*dx</pre>
$ \langle E_n \psi\rangle ^2$	<pre>def Phi(n, x):     return(np.sqrt(2/L)*np.sin(n*np.pi*x/L))</pre>
$\begin{pmatrix} \psi(\Delta x) \\ \psi(2\Delta x) \\ \psi(3\Delta x) \\ \psi(4\Delta x) \\ \vdots \\ \vdots \end{pmatrix}$	$\psi(x)$
	<code>def Phi(n, x):     return(np.sqrt(2/L)*np.sin(n*np.pi*x/L))</code>
	$dx$

# Interview Structure

## Participants...

- Described each card in their own words
- Sorted the cards in three ways
  - Initial Sort
  - Second Sort
  - Discrete and Continuous sort

# Analysis Overview

- Phenomenographical Approach
- Concept image as a theoretical framework
- Thematic analysis
  - Emergent coding scheme



PROCESS DATA



CODE DATA



LINK CODES



ESTABLISH  
THEMES

# Themes

## Discreteness of Representations

Representations are used to determine how discrete something is

## Computational Approximations

Code is recognized as a discretized approximation to the analytical case

## Separation between Particular Values

A discrete thing has particular possible values and/or separated by a particular value

## Continuity of Function vs. Continuity of Domain

The continuity of something is determined by the continuity of the function (e.g.,  $\psi(x)$ ) or the continuity of the domain

# Discreteness of Representations Examples

- Dirac and Matrix Notation
- Functions of  $x$
- Code



2<sup>nd</sup> Sort

"You definitely use **this** [group] for an infinite square well, versus **these** [groups] are best used probably for like spin systems."

Bra-ket representations	Matrix representations	Function Representations	Code Representations
$ E_n\rangle$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\int \varphi_n^*(x)\psi(x)dx$	<code>dx = 0.01</code>
$ +\rangle$		$\varphi_n(x)$	<pre>def Phi(n, x):     return np.exp(2j*pi*x*(n*pi/L))</pre>
$ \psi\rangle$	$\begin{pmatrix} \psi(\Delta x) \\ \psi(2\Delta x) \\ \psi(3\Delta x) \\ \psi(4\Delta x) \\ \vdots \\ \vdots \end{pmatrix}$	$\psi(x)$	<pre>def Psi(x):     return np.exp(1j*pi*x**2/L)</pre>
$ \langle E_n \psi\rangle ^2$		$ \psi(x) ^2$	<pre>L = 1 n = 1 sum = 0 for x in np.arange(0, L, dx):     sum += np.conj(Phi(n, x))*Psi(x)*dx</pre>
$ \langle + \psi\rangle ^2$		$dx$	4 items
$\langle + \psi\rangle$	$\vdots$	$\Delta x$	
$\langle x \psi\rangle$	$\begin{pmatrix} 1 & 0 \\ 0 & i/\sqrt{2} \end{pmatrix}$	6 items	
	3 items		

# Discreteness of Representations Examples

- Dirac and Matrix Notation
- Functions of  $x$
- Code



Jack

Disc. Cont. Sort

"I suppose **this** [ $\varphi_n(x)$  card] is maybe less, less --- because it's a function, you could talk about it in a more continuous way. So yeah, maybe that would be better **there** because it is a function of  $x$  rather than like a bracket notation or a matrix"

Discrete	Continuous
$ +\rangle$	$\int \varphi_n^*(x)\psi(x)dx$
$ \psi\rangle$	$\varphi_n(x)$
$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\psi(x)$
$\langle + \psi\rangle$	$\langle x \psi\rangle$
$(1 \ 0) \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$	$ \psi(x) ^2$
$ E_n\rangle$	$dx = 0.01$
$ \langle E_n \psi\rangle ^2$	<pre>def psi(x):     return np.sqrt(2/3)*np.sin(n*np.pi*x/L)</pre>
$ \langle + \psi\rangle ^2$	<pre>def psi(x):     return np.sqrt(30/11)**2*np.sqrt(1/11)*(x-1)</pre>
$\Delta x$	$\begin{pmatrix} \psi(\Delta x) \\ \psi(2\Delta x) \\ \psi(3\Delta x) \\ \psi(4\Delta x) \\ \vdots \\ \vdots \\ \vdots \end{pmatrix}$
	<pre>L = 1 n = 1 sum = 0 for x in np.arange(0, L, dx):     sum += np.conj(psi(n, x))*psi(x)*dx</pre>
	$dx$

9 items

11 items

# Discreteness of Representations Examples

- Dirac and Matrix Notation
- Functions of  $x$
- Code



Emily

Disc. Cont. Sort

“The way **this** is being coded, you’re not really making a continuous function in actuality, ... but I think it is trying to mimic a continuous thing, it’s just not that.”

Discrete	Continuous
<code>dx = 0.01</code>	
$\begin{pmatrix} \psi(\Delta x) \\ \psi(2\Delta x) \\ \psi(3\Delta x) \\ \psi(4\Delta x) \\ \vdots \\ \vdots \\ \vdots \end{pmatrix}$	$\varphi_n(x)$
$\Delta x$	$dx$
$ \langle +   \psi \rangle ^2$	$\psi(x)$
$ \langle E_n   \psi \rangle ^2$	$ \psi(x) ^2$
$ E_n\rangle$	
$\langle +   \psi \rangle$	
$(1 \ 0) \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$	
$ \psi\rangle$	
$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	
$ +\rangle$	

`dx = 0.01`

# Takeaways

Students use representations as a heuristic to determine discreteness

Dirac notation and matrix notation are strongly associated with being discrete

The presence of “ $x$ ” signals continuous, regardless of representation

Code is discrete or continuous, depending on if students key into the structural limitations of code or what the code represents

# Thank you!

Many thanks to the OSUPER team, our participants, and our funding supporters!

Find the slides:

Additional Questions: [solorich@oregonstate.edu](mailto:solorich@oregonstate.edu)



DUE 1836603

DUE 1836604

Find resources & research:

-  [paradigms.oregonstate.edu](http://paradigms.oregonstate.edu)
-  [osuper.science.oregonstate.edu](http://osuper.science.oregonstate.edu)

See PERC poster:



# OSU is hiring!

**Search has begun for tenure-track positions  
in PER**

Contact Search Committee Chair Dr. Ethan Minot for details

[Ethan.Minot@oregonstate.edu](mailto:Ethan.Minot@oregonstate.edu)

[osuper.science.oregonstate.edu](http://osuper.science.oregonstate.edu)



**Oregon State**  
**University**

# Code Snippet Sorting Examples

```
L = 1
n = 1

sum = 0
for x in np.arange(0, L, dx):
    sum += np.conj(Phi(n, x))*Psi(x)*dx
```

```
def Phi(n, x):
    return(np.sqrt(2/L)*np.sin(n*np.pi*x/L))
```

```
def Psi(x):
    return(np.sqrt(30)/L**2/np.sqrt(L)*x*(x-L))
```

```
dx = 0.01
```

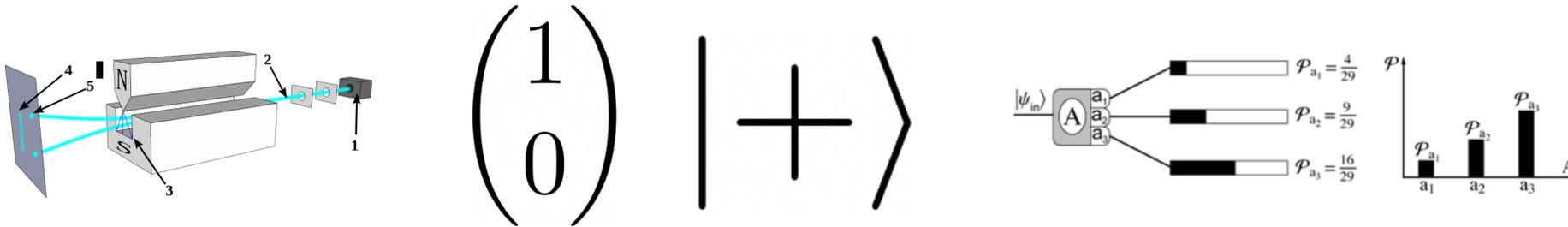
## Definition

# Concept Image

- Describes the “total cognitive structure that is associated with [a] concept” [4]
- Builds up over time through experiences (and can change)
- Different stimuli can activate different parts of one’s concept image
  - Does not need to be coherent

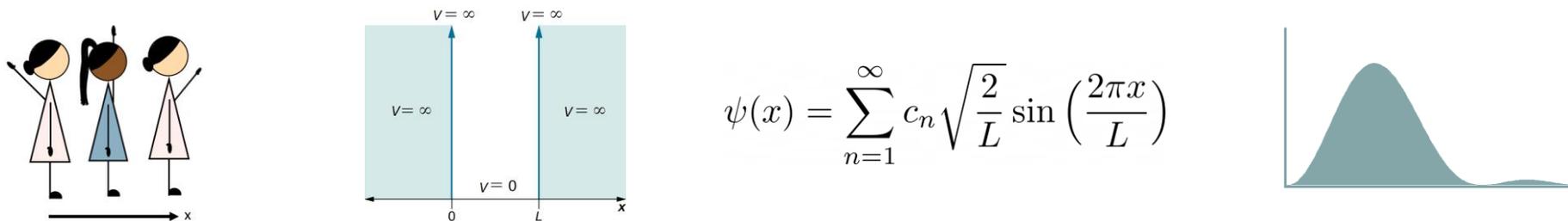
# PH 425 Course Structure

Introduce QM with spin-1/2 systems



Discrete systems

End with the particle in a box



Continuous system