Discrete and Continuous Connections: Reflective Interview on a Computational Activity

Christian D. Solorio (he/him)
Elizabeth Gire
Oregon State University
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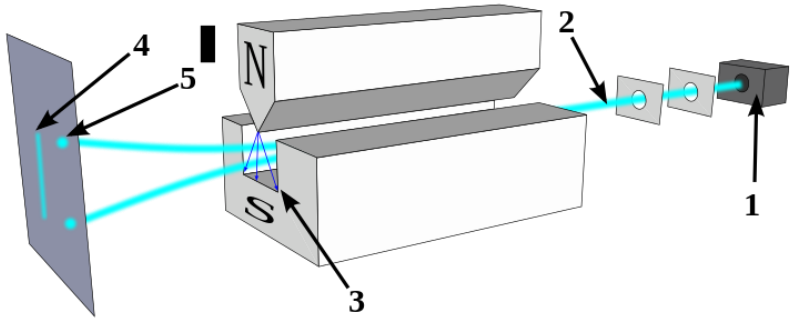
DUE 1836603
DUE 1836604

Broad Research Goal

Investigate how students leverage what they know about discrete quantum systems when learning about continuous ones

Spins First Quantum Mechanics Course Overview

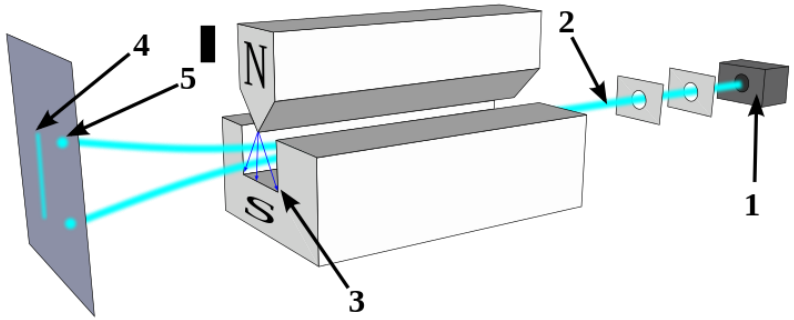
Spin-½ Systems



- Use matrices and linear algebra
- States are discrete, finite

Broad Spin-1/2 Course Overview

Spin-½ Systems



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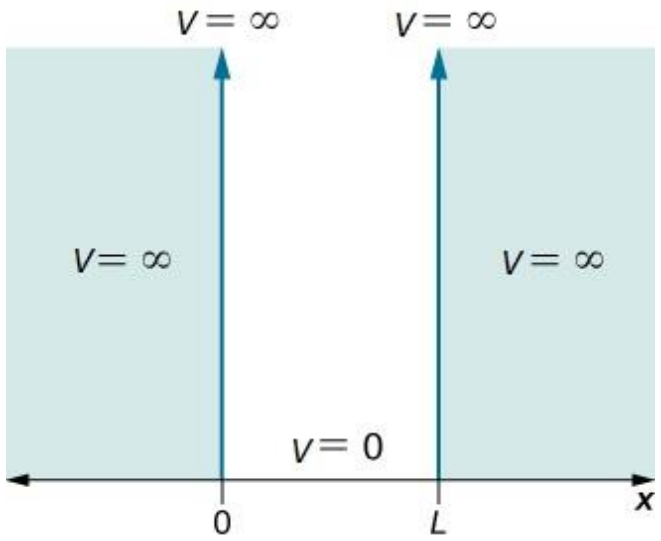
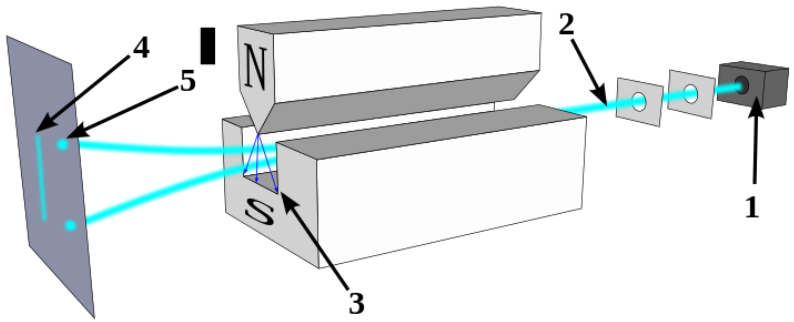
Infinite Square Well



- Uses linear algebra and calculus
- States can be continuous (position basis), discrete and infinite (energy basis)

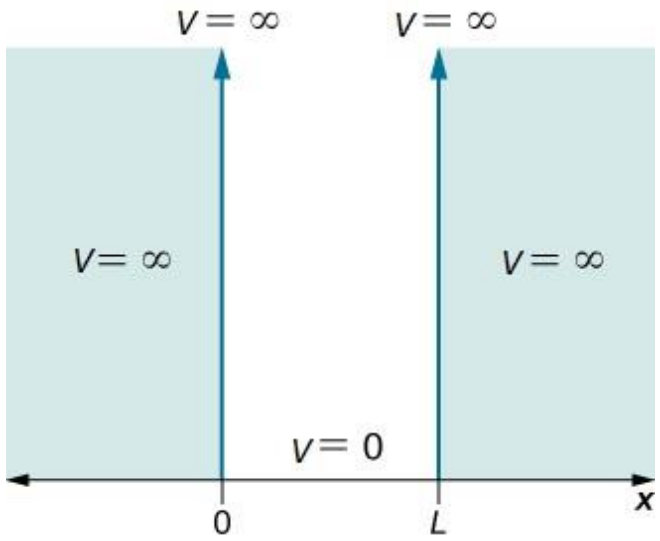
Broad Spin-1/2 Course Overview

Spin-½ Systems



Difficult transition

Infinite Square Well



Broad Spin-1/2 Course Overview

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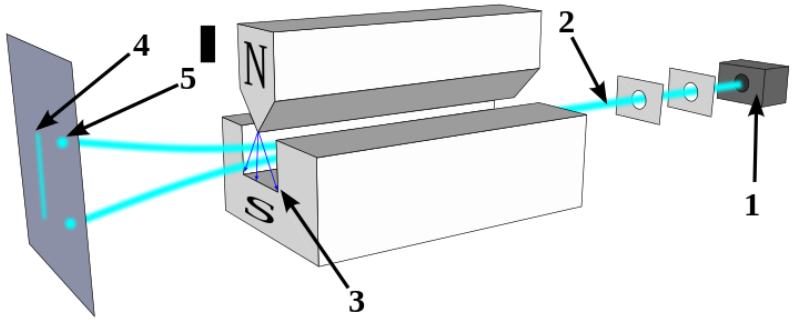

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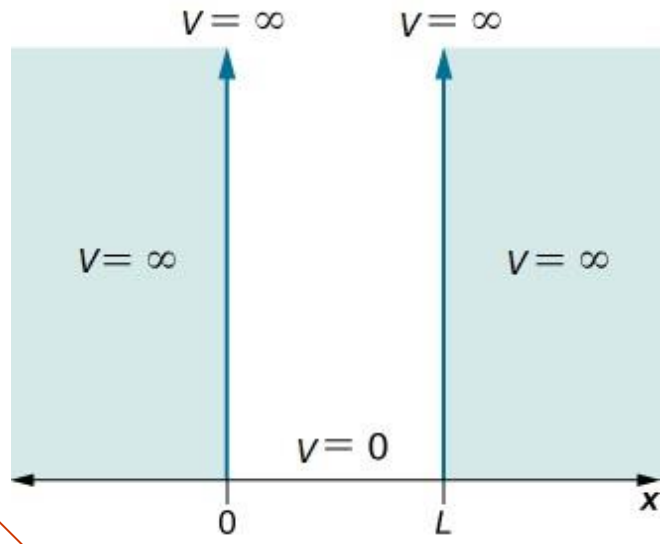
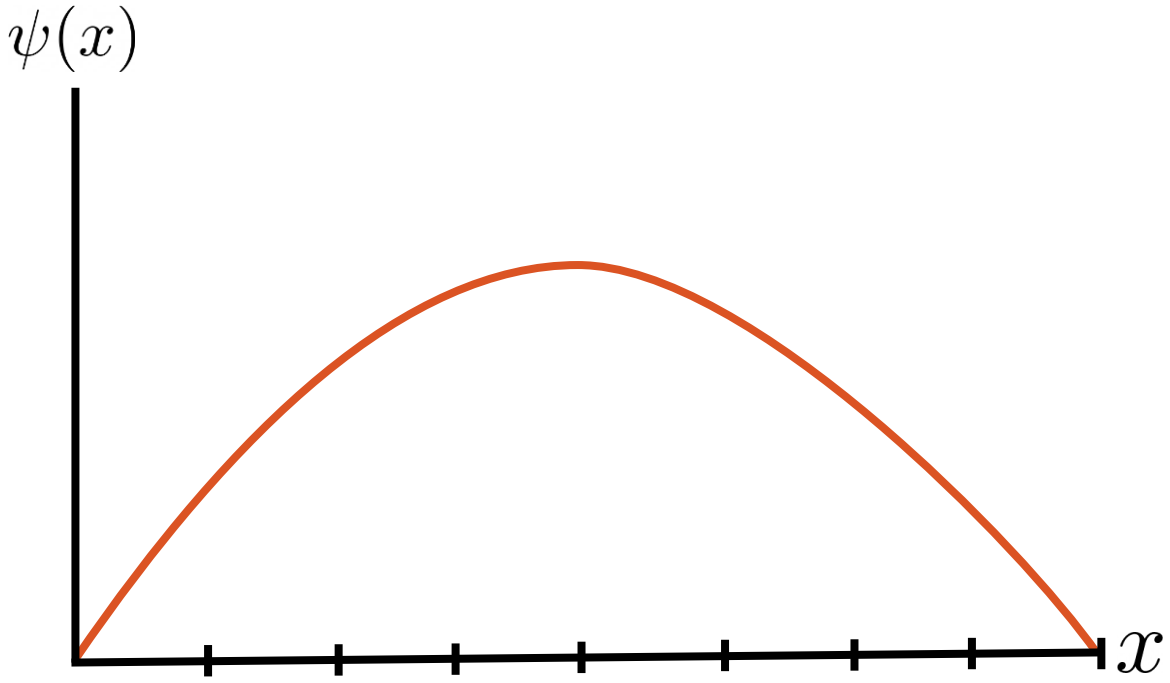


- Wider variety of mathematics are required
- Trouble with Dirac Notation (e.g., $|\psi\rangle$ vs $\psi(x)$)
- Inner products *look* different
- We don't know what connections students *are* making

Utilizing Computation as a Bridge

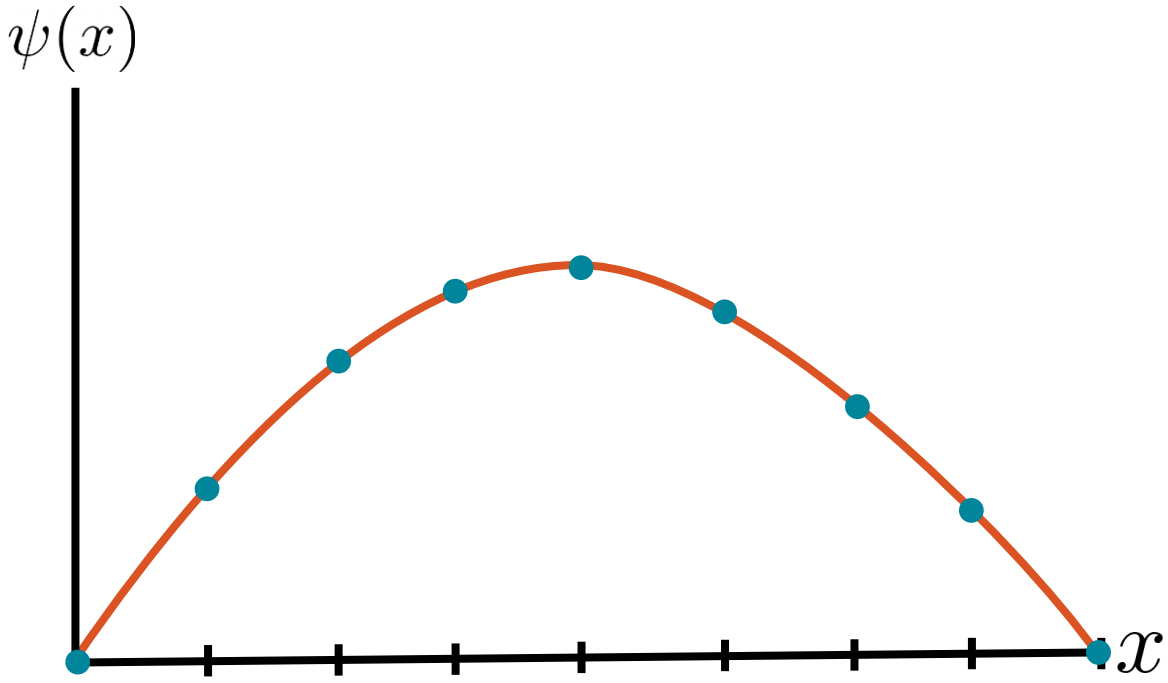
- Wave functions are discretized for computational operations



$$\psi(x) = Ax(L - x)$$

Discretization!

- Wavefunctions are discretized for computational operations



$$\psi(x) = Ax(L - x)$$

$$\psi(x) \rightarrow \{\psi(0), \psi(\Delta x), \psi(2\Delta x), ...\psi(L)\}$$

Research Question

What are the conceptual challenges of discretization that students engage with in a computational lab course?

Video Elicitation Interview

Mary was a part of an observational study in W21

Became a TA for the computational lab course in W22

Conducted an hour long semi-structured interview

Watched a 10-min clip of her and her partner working on a computational activity

Mary reflected on her experience of the activity as a student and as a TA

Activity: Finite Difference Approximation of $\hat{T}$

$$\begin{pmatrix} \hat{T}\psi(\Delta x) \\ \hat{T}\psi(2\Delta x) \\ \hat{T}\psi(3\Delta x) \\ \hat{T}\psi(4\Delta x) \\ \vdots \end{pmatrix} = \frac{\hbar^2}{2m\Delta x} \underbrace{\begin{pmatrix} 2 & -1 & 0 & 0 & \dots \\ -1 & 2 & -1 & 0 & \dots \\ 0 & -1 & 2 & -1 & \dots \\ 0 & 0 & -1 & 2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}}_{\text{KE Operator}} \underbrace{\begin{pmatrix} \psi(\Delta x) \\ \psi(2\Delta x) \\ \psi(3\Delta x) \\ \psi(4\Delta x) \\ \vdots \end{pmatrix}}_{\text{Discretized Wave Function}}$$

Activity: Finite Difference Approximation of $\hat{T}$

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Results:

Two Productive Discretization Challenges

- Recognizing functions as column vectors
- Interpreting Δx

Challenge 1: Recognizing functions as column vectors

Interview

"It's hard to think about what it even means to have the **wave function** in a **matrix form** and like have an operator in matrix form"

Mary (W22)

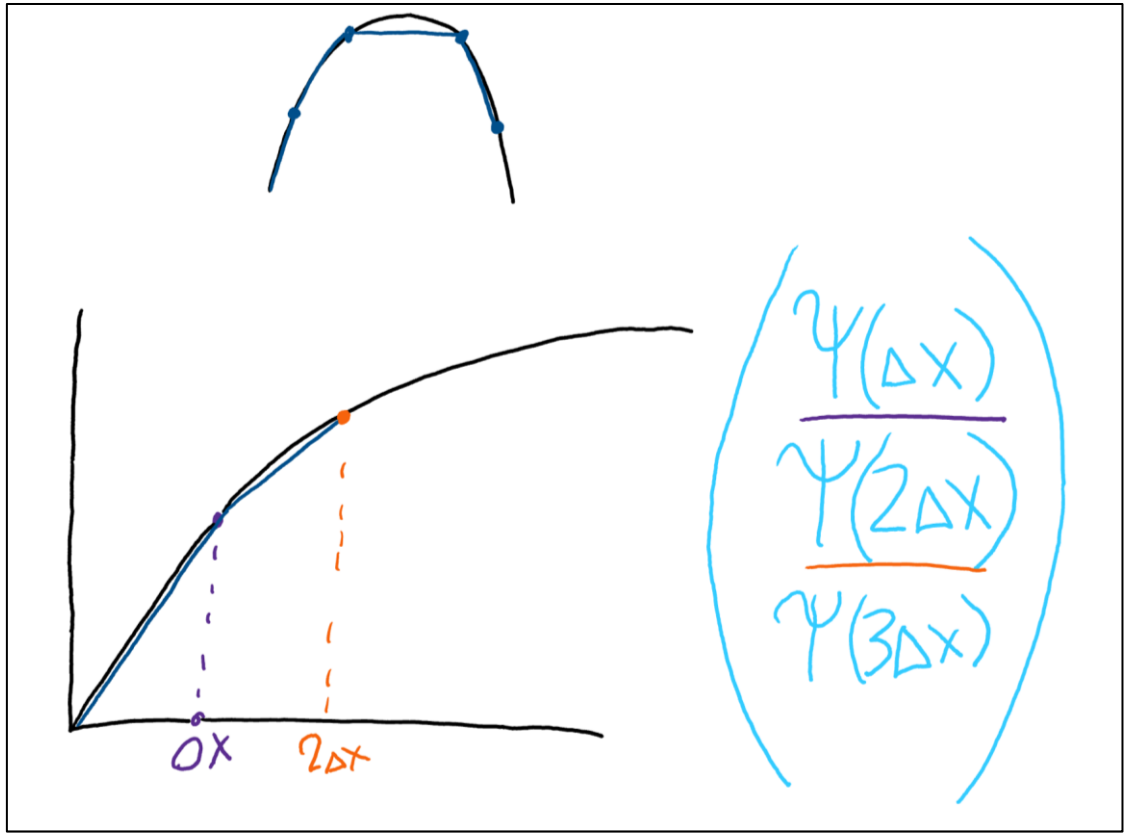
Challenge 1: Recognizing functions as column vectors

+ Understanding that ψ is an uncountably infinite set of coefficients

+ Relationship between Δx size and accuracy of approximation

$$\psi(x) \longrightarrow \begin{pmatrix} \psi(\Delta x) \\ \psi(2\Delta x) \\ \psi(3\Delta x) \\ \psi(4\Delta x) \\ \vdots \end{pmatrix} \longrightarrow \psi(x) \rightarrow \{\psi(0), \psi(\Delta x), \psi(2\Delta x), \ldots \psi(L)\}$$

Challenge 1: Recognizing functions as column vectors



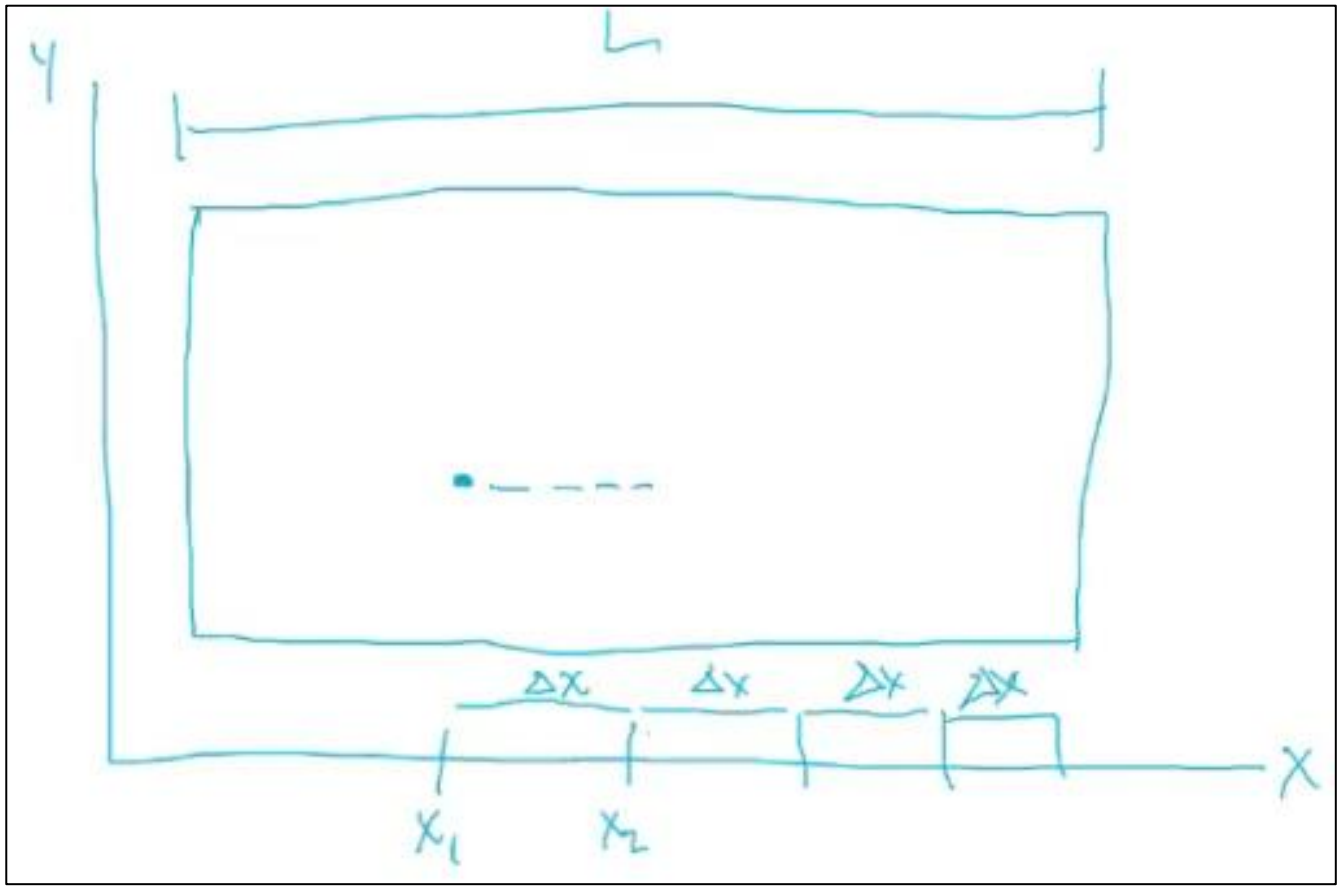
Mary's drawing of the discretized wave function from the W22 Interview

Interview

"Yeah, so we're **approximating a continuous psi**…with a discrete representation and **rather than having some infinitely long set** from… 0 to L…we're breaking it up into — **we're only taking certain points along the line**."

Mary (W22)

Challenge 2: Interpretations of Δx



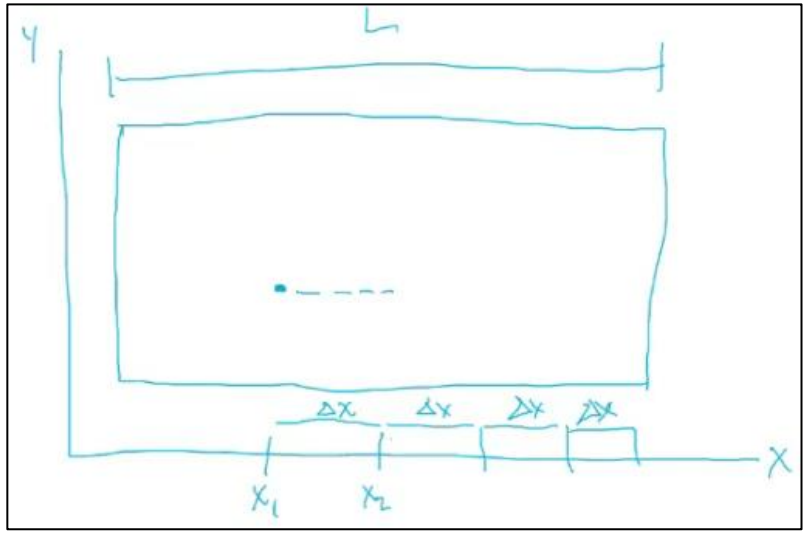
Mary's partner's drawing of the infinite square well from the W21 video

Interview

"I think just seeing Δx on a page is nice, because then you're like, 'Okay, what was Δx supposed to be?'. Um, except for that what I'm learning now is that **Δx isn't actually like on a bar graph** where you have between two points [sic]. It's literally **just a point on the graph**."

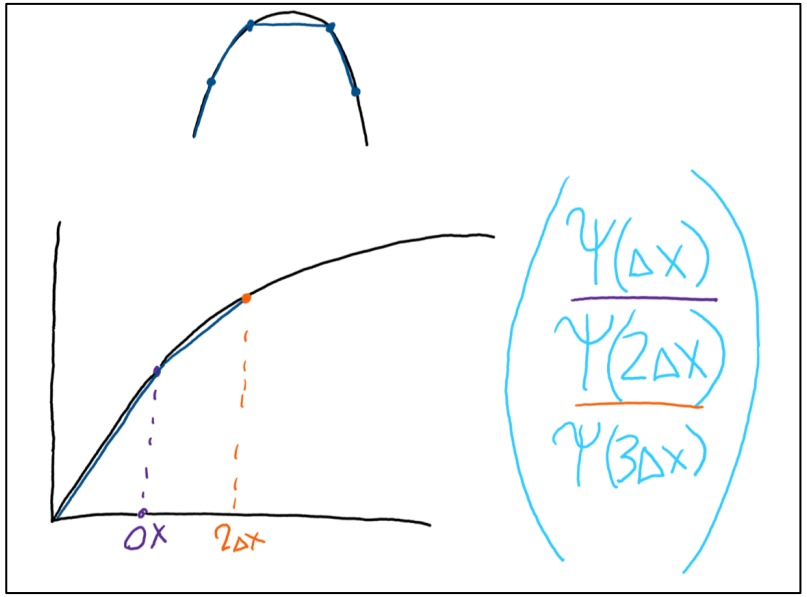
Mary (W22)

Challenge 2: Interpretations of Δx



"Space between points" interpretation

- Understanding how Δx breaks up L
- Connection between Δx size and number of ψ elements



"Single Point" interpretation

- Evaluating $\psi(\Delta x)$
- Connecting $\psi(x)$ to a graphical representation of the ISW

Conclusions

- Discretization can be conceptually challenging
- The challenges encouraged productive reasoning
- Activities like this may illuminate discrete and continuous connections

Instructional Implications

- Students may benefit from additional instruction on the connections between functions and vectors
- Terms like Δx carry contextual meaning
- Access to multiple representations can help students better understand discretization

Thank you!

Many thanks to the OSUPER team, our participant, and our funding supporters!

Additional Questions: solorich@oregonstate.edu

Find resources & research:

paradigms.oregonstate.edu

osuper.science.oregonstate.edu

Try the Activity:

https://beav.es/TQP

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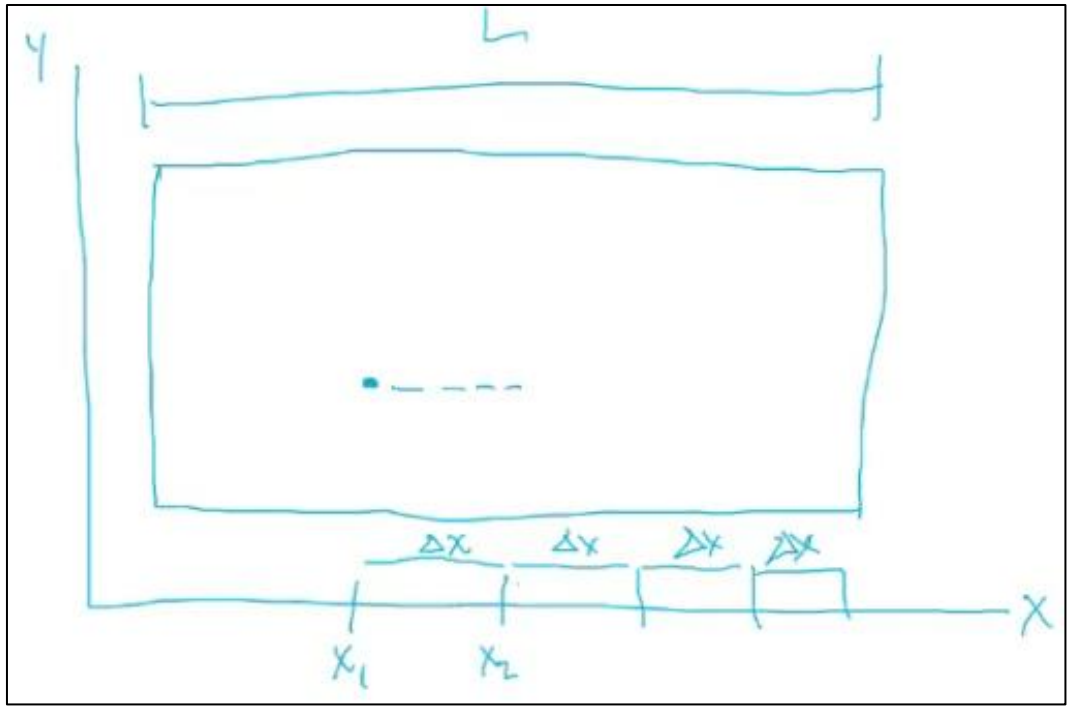
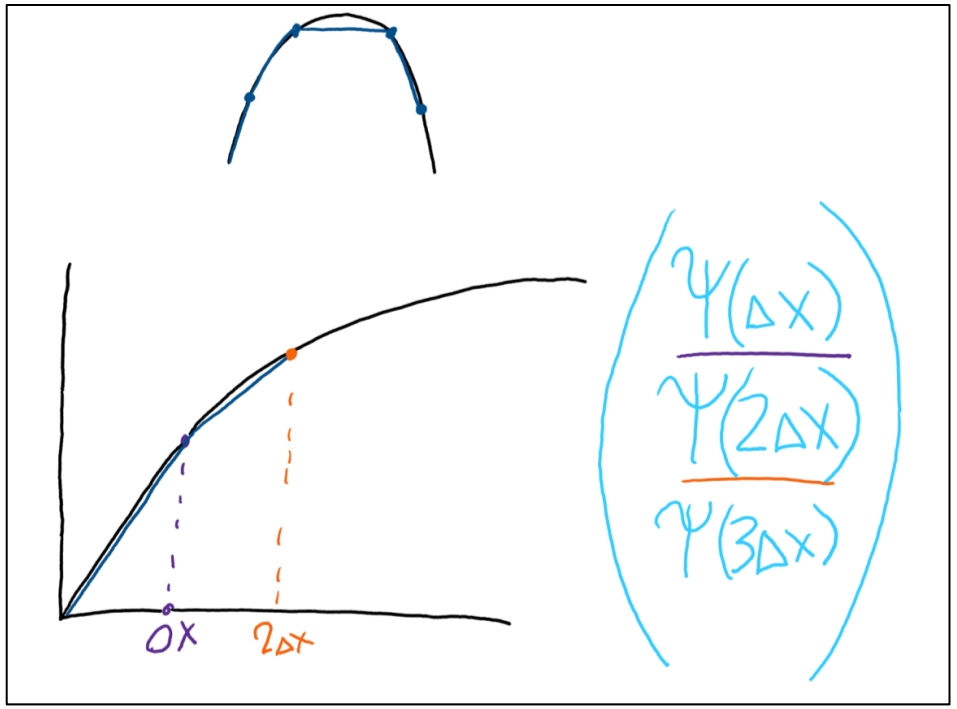
Graphical Representation Comparison



Mary's partner's drawing in W21 video



Mary's drawing during W22 interview