Quantum Mechanics Students' Understanding of "Discrete", "Continuous", and Computational Approximations

Christian D. Solorio (he/him)
Elizabeth Gire
David Roundy
Oregon State University



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Sorry!

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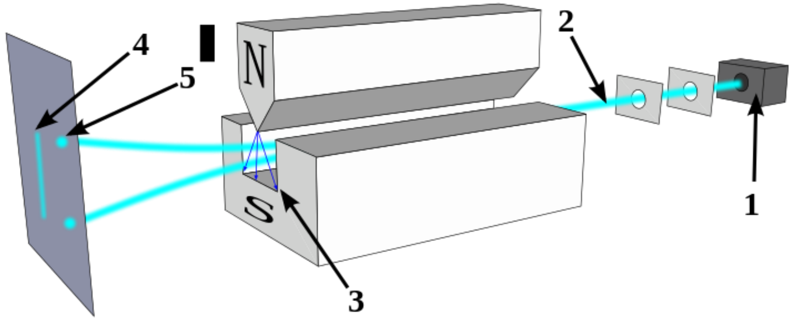
Broad Spin-1/2 Course Overview

Spin-½ Systems



- Use matrices and linear algebra
- States are discrete, finite

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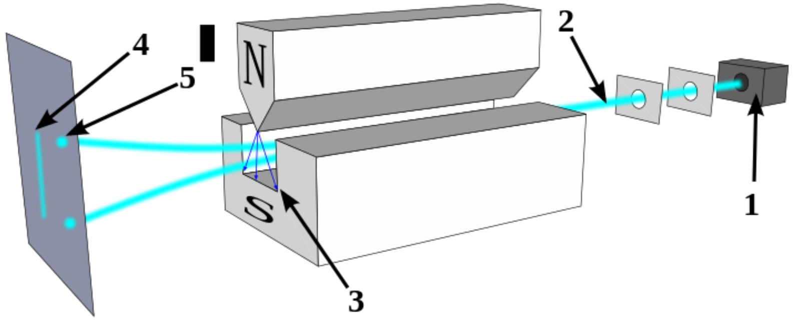
Spin-½ Systems



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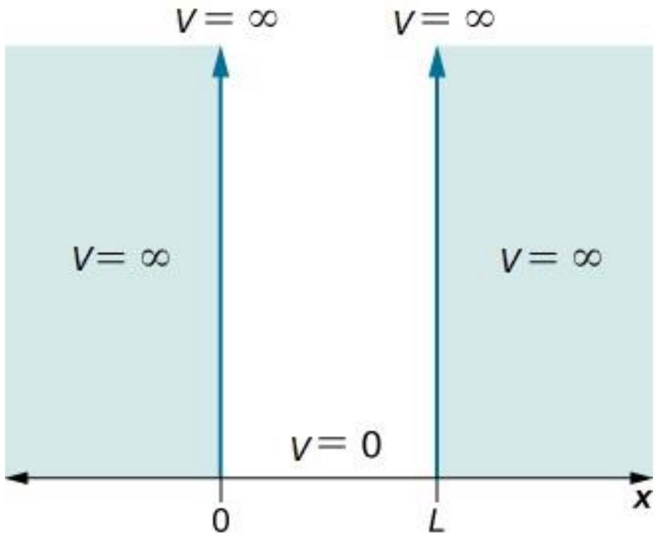
- States are discrete, finite

Infinite Square Well



- Uses linear algebra and calculus
- States can be continuous (position basis), discrete and infinite (energy basis)

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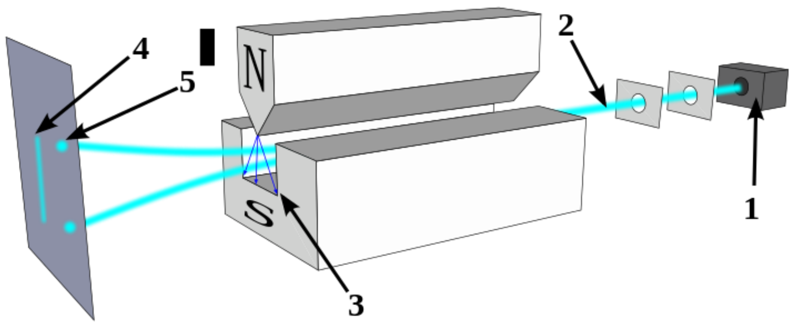
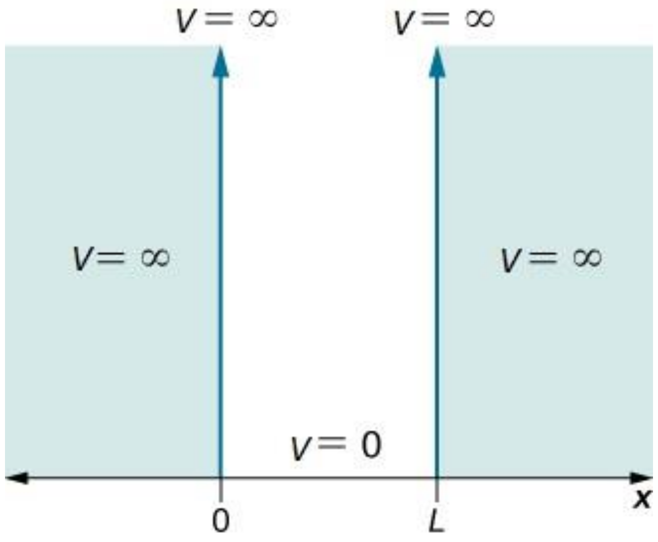
Spin-½ Systems



Difficult transition

Infinite Square Well



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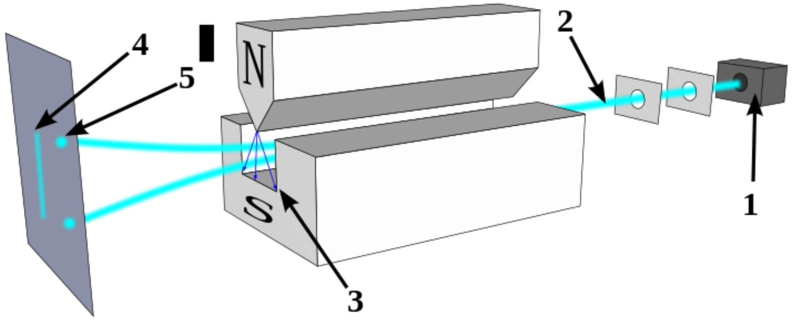

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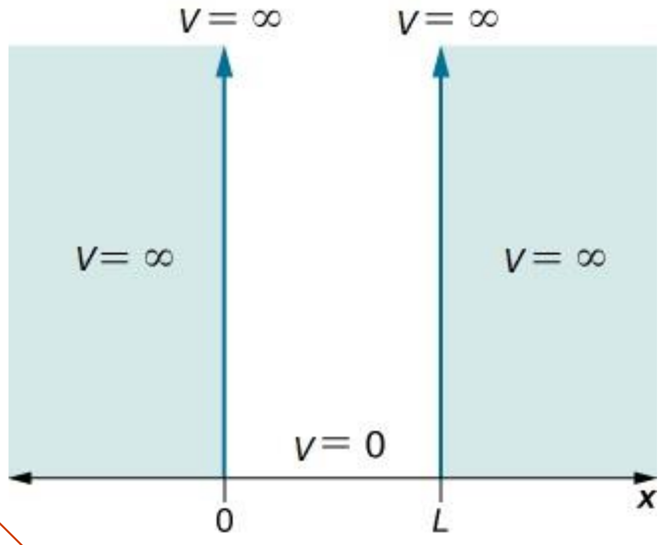


- Wider variety of mathematics are required
- Trouble with Dirac Notation (e.g., $|\psi\rangle$ vs $\psi(x)$)
- Inner products *look* different
- We don't know what connections students *are* making

Research Questions

1 How do students understand “discrete” and “continuous” in QM?

2 What connections do students make between discrete and continuous?

Analysis Overview

- Phenomenographic Approach
- Concept image as a theoretical framework
- Thematic analysis

PROCESS DATA

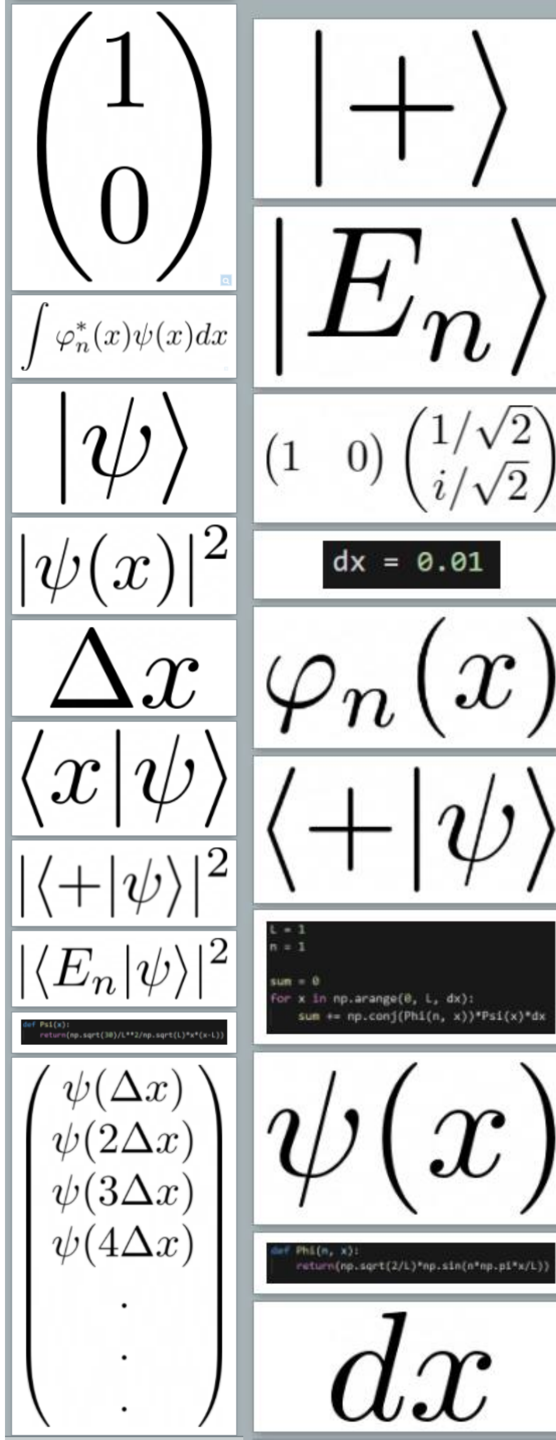
CODE DATA

LINK CODES

ESTABLISH THEMES

Card Sorting Task Interview Methods

- Conducted remote semi-structured interviews with 6 participants
- Participants followed a *think aloud* protocol
- Organized 20 quantum mechanics cards
- Used an online card sorting application “OptimalSort”



Card Sorting Interview Structure

Participants…

- Described each card in their own words
- Sorted the cards in a few ways
  - Discrete and Continuous sort

Phenomenon: Continuity in Quantum Mechanics

Understanding of Discrete

Particular values
Separation of value

Understanding of Continuous

Continuity of the domain
Continuity of the function
Unboundedness

Discreteness of Representations

Bracket + Matrix notation signal discrete
Functions signal continuous

Computational Approximations

Step size and accuracy
Integration as a discrete sum

Phenomenon: Continuity in Quantum Mechanics

Understanding of Discrete	Understanding of Continuous
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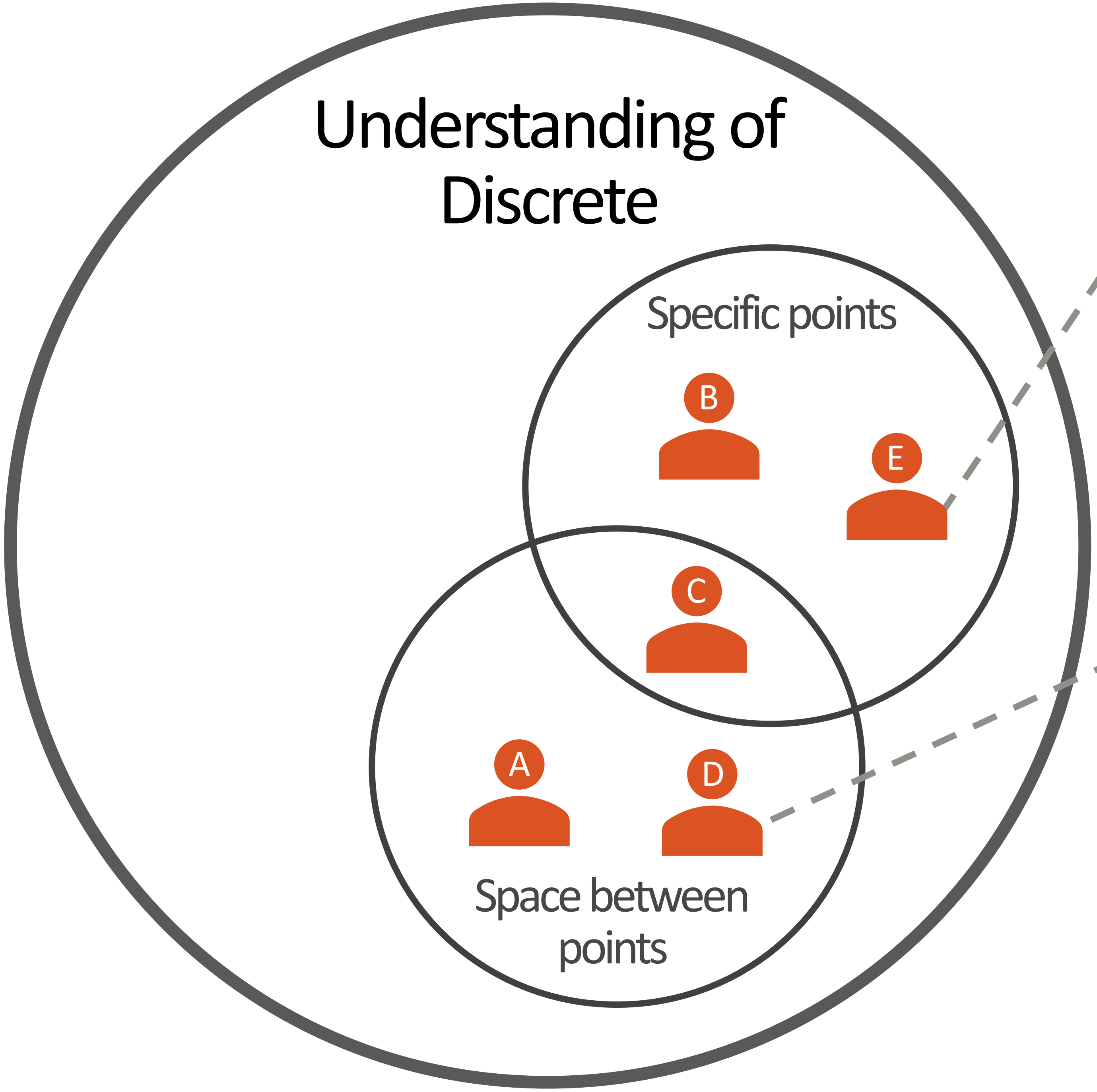
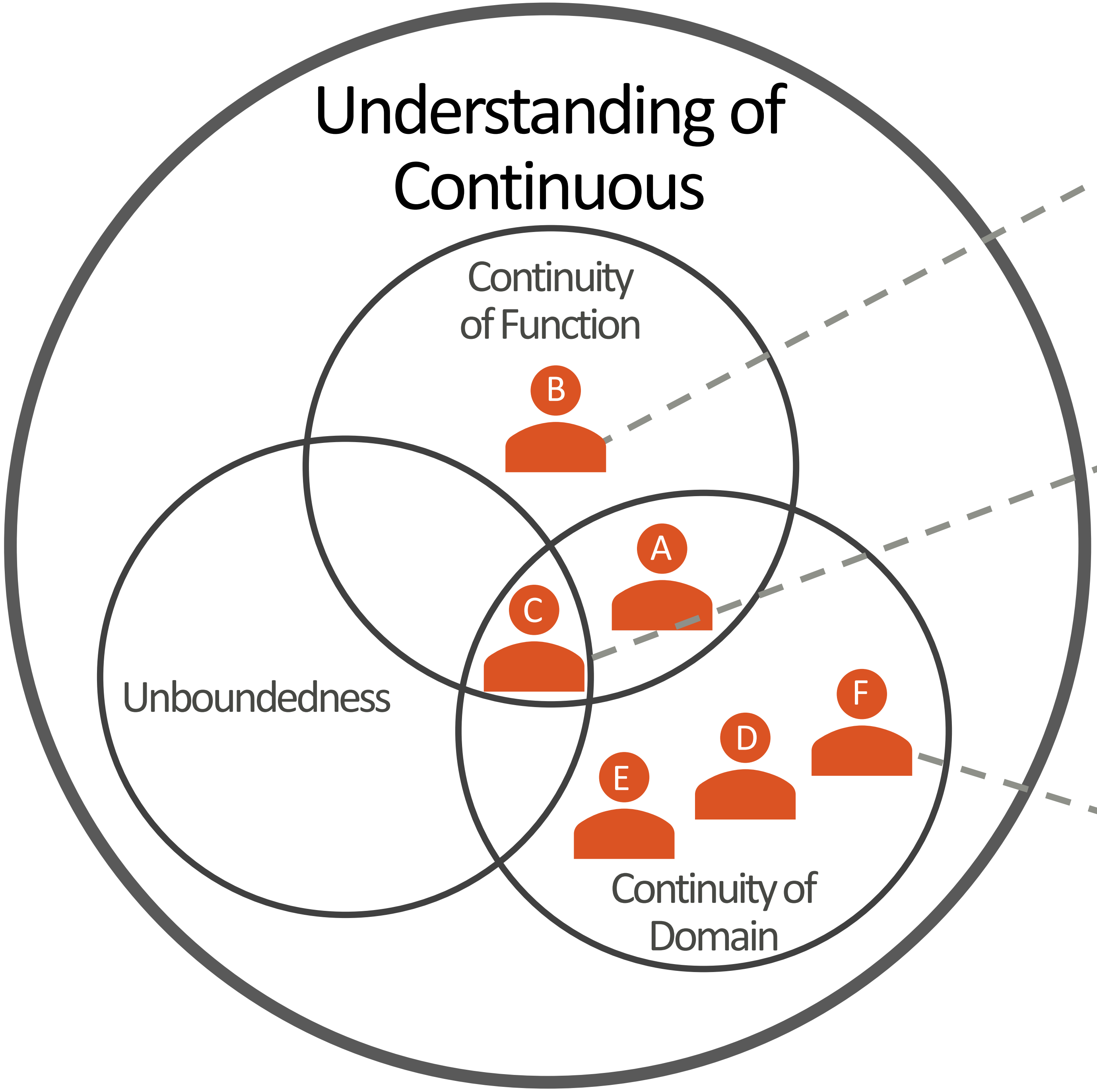
Discreteness of Representations	Computational Approximations
Bracket + Matrix notation signal discrete Functions signal continuous	Step size and accuracy Integration as a discrete sum



"**Discrete** would be, like there's **only certain values possible**. Like you can only get spin up or spin down, there's **no in-between**. You can only get, like energy comes in those packets. You can only get discrete amounts of energy each time."

"I guess you could think of something — er, continuous stuff as a spectrum of things and **discrete stuff** as like **belt notches**, I don't know. Like you can't use the **space between** notches on your belt, you're not going to be able to put on your belt. You've gotta put the little metal thing on your belt into one of the holes"



"I know discrete and continuous, that will **relate to the functions** — whether or not we see a **continuous curve**, you know, mathematically speaking, one which doesn't have points, doesn't suddenly drop, or rise, no asymptotes.

"Well just because [$\langle E_n|\psi\rangle$] is continuous meaning like you can **keep plugging numbers in for n for as long as you want**. I mean, at least theoretically, I think. Because they represent eigenstates, I don't know if that you can necessarily plug in 40 million as your n number realistically or experimentally, but they analytically are continuous."

"So, what I did was, for like the continuous, I just put what had **some kind of function of x**. Normally when you have that, you're going to be working with **position** which is a continuous thing"

Takeaways

- Students carry a variety of understandings of discrete and continuous
- Words like “continuous” carry contextual meaning
  - One aspect of someone’s personal concept image can confuse them
- **Teaser:** Computation offers students an opportunity to bridge ideas about discrete and continuous via Riemann sums
  - Riemann sums were particularly valuable in participants’ reasoning about discretely approximating integrals

Oregon State University

Thank you!

Many thanks to the OSUPER team, our participants, and our funding supporters!

Find the slides:

Additional Questions: solorich@oregonstate.edu

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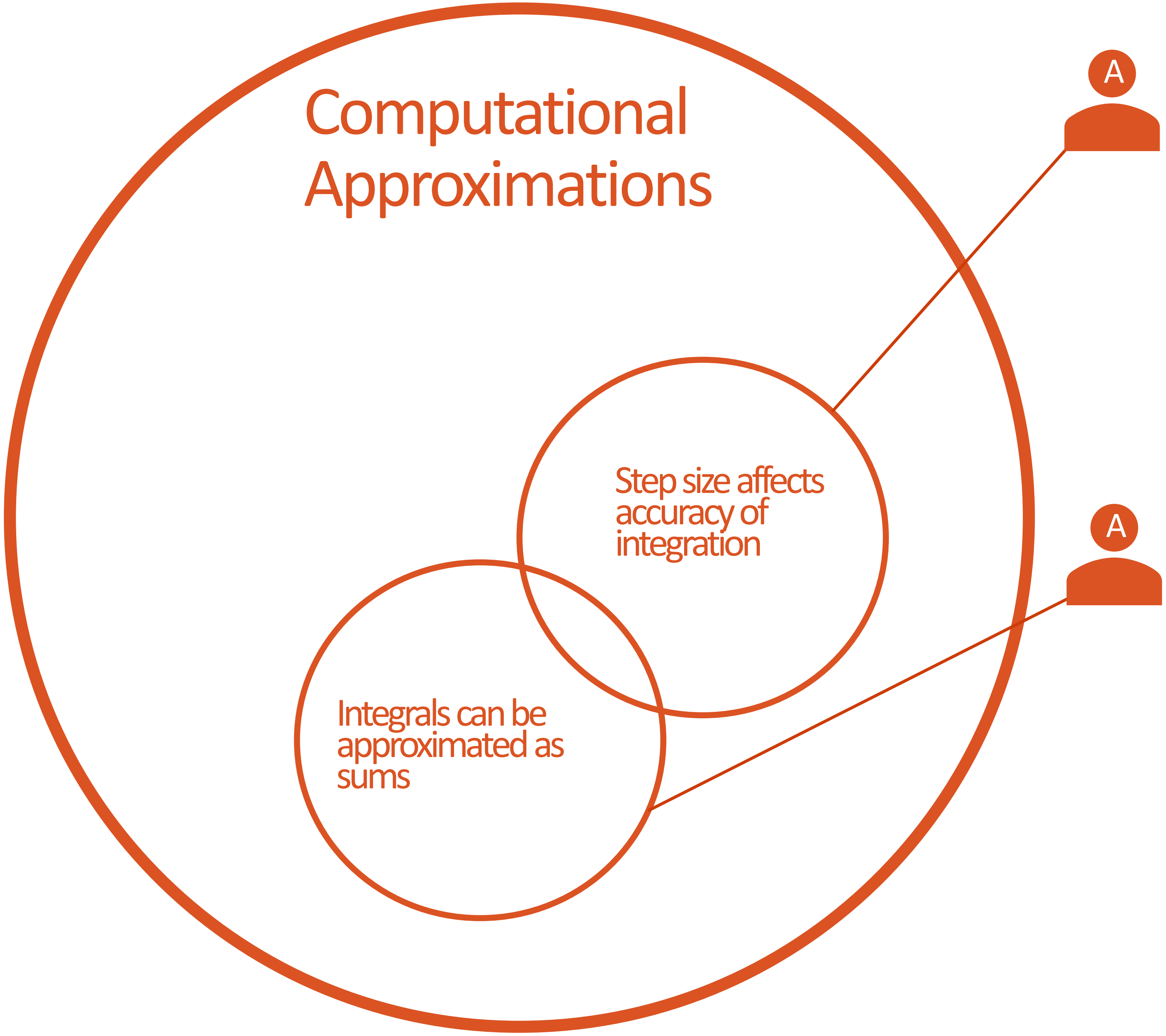
Find resources & research:
paradigms.oregonstate.edu
osuper.science.oregonstate.edu

Try the Activity:
https://beav.es/5DG

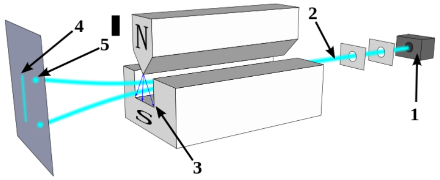


"In code, you have to be really explicit and you have to define certain things, have exact values, and like **running a for loop isn't the same as an integral** because an integral, you get so close to continuous that you basically are. Like your infinitesimals can be infinitesimally small, versus like these are — you're **chunking** things."

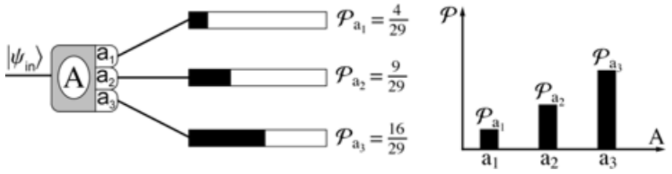
"If this integration represents a **bunch of little sums**, like when you're doing an integral that's kind of what you're **doing in the background.** You are finding some function that would represent that net change or whatever, or, if the answer from the integral is a function. But if it's just a number, like a definite integral, yeah you are working out a way to find that net change — that sum — without doing all those little sums painstakingly.

PH 425 Course Structure

Introduce QM with spin-1/2 systems

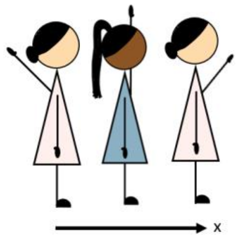


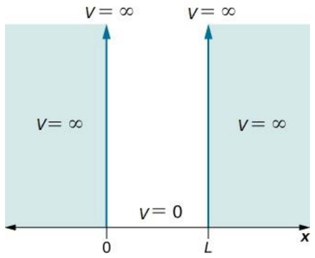
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |+\rangle$$



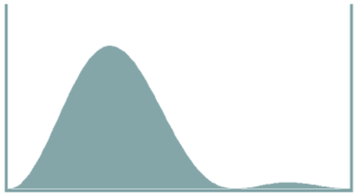
Discrete systems

End with the particle in a box





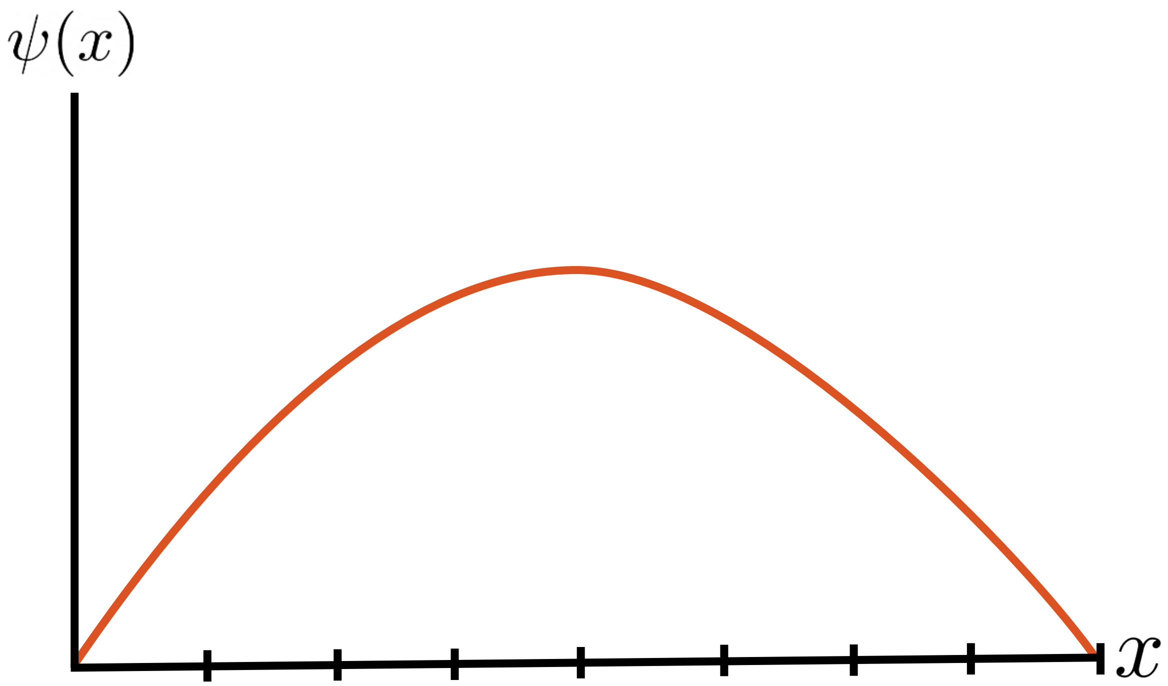
$$\psi(x) = \sum_{n=1}^{\infty} c_n \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$$



Continuous system

Utilizing Computation as a Bridge

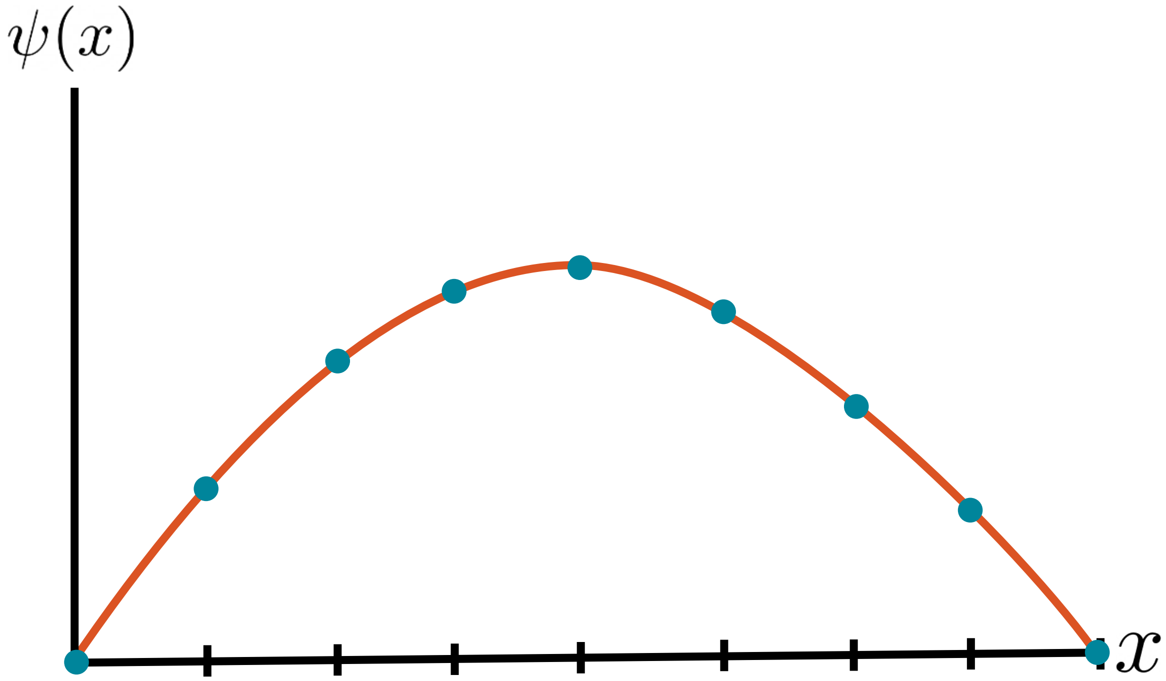
- Wavefunctions are discretized for computational operations



$$\psi(x) = Ax(L - x)$$

Discretization!

- Wavefunctions are discretized for computational operations



$$\psi(x) = Ax(L - x)$$

$$\Downarrow$$

$$\psi(x) \rightarrow \{\psi(0), \psi(\Delta x), \psi(2\Delta x), ...\psi(L)\}$$

Computational Lab Course Structure

- Supports physics learning in the context of programming
- Pair-programming in Python
- Taught by an instructor and a team of TAs
- Wavefunctions focused approach

Code Snippet Sorting Examples

```
L = 1
n = 1

sum = 0
for x in np.arange(0, L, dx):
    sum += np.conj(Phi(n, x))*Psi(x)*dx
```

```
def Phi(n, x):
    return(np.sqrt(2/L)*np.sin(n*np.pi*x/L))
```

```
def Psi(x):
    return(np.sqrt(30)/L**2/np.sqrt(L)*x*(x-L))
```

```
dx = 0.01
```

Definition

Concept Image

- Describes the "total cognitive structure that is associated with [a] concept" [4]
- Builds up over time through experiences (and can change)
- Different stimuli can activate different parts of one's concept image
  - Does not need to be coherent

Tall and Vinner, Educ. Studies in Math **12,** 151-169 (1981)

Definition

Phenomenography

- Research methodology
- "Second order" perspective
- Seeks to identify the salient variations that comprise a collective experience

Marton, Instructional Science **10**, 177 (1981)