Explicitly prompting covariational reasoning in a thermodynamics context

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Making sense of the relationships between interdependent variables is an important skill in physics. We developed in-class activity in the junior-level thermodynamics course to help students with this covariational reasoning. The students worked in groups, and were provided contour maps and plastic models representing thermodynamic states of water vapor. We consider a sequence of prompts from the activity. We are interested in when and why the students chose to use a particular representation in response to the prompts, and how the prompts elicited covariational reasoning. We find that the students preferred representations that were more straightforward to interpret. Further, we find that the prompts elicited some direct statements about the codependent nature of thermodynamic variables.

I. INTRODUCTION

Many aspects of thermodynamics are challenging to learn, including the fact that the variables are not spatial, the variables cannot be treated as orthogonal, the variables are interrelated, and the graphs representing these multiple variables are complicated. These aspects make thermodynamics an interesting context for studying students' use of representations and students' reasoning about covariation among variables.

Mathematics education research has a large body of literature on student understanding of covariation (*e.g.,* Refs. [1–3]), finding that even in straightforward applications, covariational reasoning is difficult for many students. We draw on the definition of covariation reasoning by Carlson *et. al*: "the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other" [4]. However, because thermodynamics involves systems with two (or more) independent variables, multivariable covariational reasoning involves coordinating and attending to *three* (or more) varying quantities.

This study is part of a larger study within the *Raising Physics to the Surface* project, regarding student use of surfaces for thermodynamics. The project aims to develop curriculum to help students understand professional representations in physics using 3D plastic graphs ("surfaces"). Research on representational fluency and students' use of multiple representations has shown that they may need explicit instruction in representation use [5–7], they should be taught the value of different representations [8, 9], and their investigations should be constrained in useful ways [10].

Our study focuses on investigating an activity designed to help students with covariation in thermodynamics, which has been informed by results from other aspects of the project [11]. Our research question questions are:

RQ1: "*How did the activity prompts elicit covariational reasoning?*"

RQ2: "*How did the students select a graph in response to the activity prompts?*"

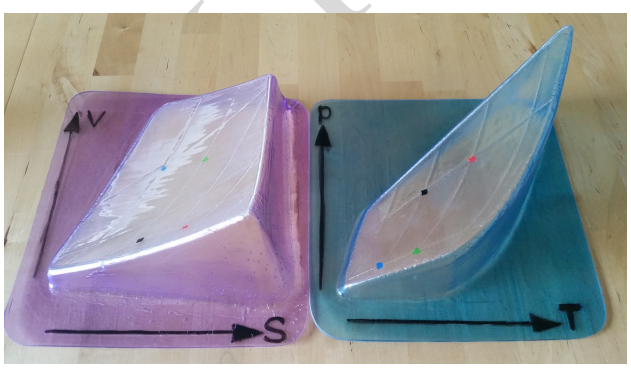

FIG. 1. The surfaces provided to students (axis labels emphasized). The height of the purple (blue) surface represents U as a function of S and V (p and T) and includes contours for T and p (S and V).

II. METHODS

Video was collected from a junior-level thermodynamics course at Oregon State University in 2018. As part of instruction, the students were given a worksheet, two surfaces representing the same set of states for water vapor (Fig. 1), and contour maps matching each surface (Fig. 2). This activity was likely most students' first encounter with the plastic surfaces. Each surface also has contours etched into them, but these were not explained to the students. As can be seen in Fig. 2, the contour maps themselves had no variable labels or numbers on them, but were in color.

The worksheet begins with a prompt intended to orient the students to the representations and the five thermodynamic variables (Pressure, p; Entropy, S; Temperature, T; Internal Energy, U; and Volume, V.):

> **Orient:** *The surfaces represent measurements on a kilogram of water vapor in a piston (a graduated cylinder with a moveable top). The purple surface graph is $U(S, V)$ and the blue surface graph is $U(T, p)$. Each surface graph has a corresponding contour plot.*
>
> *Identify which contour plot goes with which surface. Label the axes of the contour plots.*

After the *Orient* prompt, the students were given the following prompts in sequence:

> **Explore:** *Starting at the red star, as you increase the volume of the system, what happens to the internal energy of the water vapor?*
>
> *Is it possible to change the volume without changing the internal energy?*

The *Explore* prompts are intentionally ambiguous to produce possible tension between students answers to the pair of prompts. During the instructors' pre-class planning meeting, the instructional team agreed that students were likely to

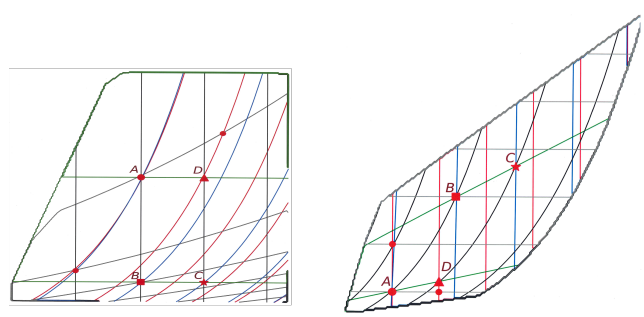

FIG. 2. The contour maps provided for these two surfaces. There are five color-coded contours, specifying the five variables S, V, U, T, and p. When given to the students, both maps were in the same dry-erase sleeve, one on the front and one on the back.

FIG. 3. Students in the Blue group (left to right: Ocean, River, Skye) after drawing the path in response to the first *Explore* prompt.

FIG. 4. Students in the Green group (left to right: Hunter, Laurel) as they discuss the first *Explore* prompt.

choose a path parallel to the volume axis along which volume increases and internal energy decreases, which was the most common answer on the pretest [11]. The second *Explore* prompt was intended to help students realize that the volume can change in many different ways.

The last analyzed prompt from the worksheet is:

> **Representation:** *Which surface/contour map did you choose to answer the questions above? Why? Can you use the other surface/contour map?*

We filmed eight students in three groups as they worked on the activity. Each student is identified by a pseudonym appropriate to the group color. The Green group had two students (Hunter and Laurel), the Red group had three students (Blaze, Phoenix, and Valentine), and the Blue group had three students (Ocean, River, and Skye). Then, portions of these videos were transcribed to include spoken words and relevant actions. Finally, we performed a Thematic Analysis [12] for each research question separately. First, we found and interpreted any statement or action that indicated the utility of one of the representations. Then, we found and interpreted any explicit statement about how one or more variables change when some other variable or variables are changed. These interpretations were used to identify themes in the data across the three groups.

III. RESULTS

We identified four predominant themes in the data: three related to RQ1 and one related to RQ2. We describe each theme alongside evidence about that theme from all three groups.

A. Considering ΔU along a path parallel to the V axis

As anticipated, all three groups chose the same path parallel to the V axis in response to the first *Explore* prompt. The Blue group actually drew an arrow (see Fig. 3) on the purple surface and stated that as V increases, U goes down. The following is evidence of this choice:

Ocean: "Right here [*pointing at red dot on purple surface*]. Volume is increasing this way [*gestures along purple surface.*]. So it [U] decreases?"

Skye: "Oh, you're right. [...] So that would be [*begins drawing on surface while looking over it*], let's try to make a straight line [*draws while making a sound effect*]. So that, yeah, that looks like it's decreasing."

The Green group reasoned similarly, although they did not draw on their surface at this point in the activity. Hunter indicated several times that as V increases, U decreases. Fig. 4 shows the Green group during the following discussion:

Laurel: "I think that intern ... [*places hand on surface*] I mean U ... this whole function [*slides finger along purple surface.*]"

Hunter: "Yeah, is U."

Laurel: "So, you can see, I mean, it moves down [*moving pencil along surface*]."

The students in the Red group also chose the path parallel to the V axis. Phoenix said, "So as you increase it [V] you move in that direction." The students decided that as V increases, U decreases. Furthermore, Phoenix made several similar statements throughout the video indicating that V only increases in one direction on the surface:

Phoenix: "So, if we say that [*picks up purple surface and holds it sideways while gesturing*] we're gonna change the volume, that means we're gonna move up or down on the ... along this axis [*gesturing along V axis as in Fig. 5*] which is the volume axis, right?"

We see that the first *Explore* prompt, then, resulted in all three groups concluding that the internal energy goes down as the volume increases.

FIG. 5. Students in the Red group (left to right: Blaze, Phoenix, Valentine) as they discuss the first *Explore* prompt.

B. Acknowledging S is held fixed along a path parallel to the V axis

Although the students did not choose any paths other than the path parallel to the V axis in response to the first *Explore* prompt, some students recognized that S is fixed along this path. In the Blue group, River said early on that "entropy is constant" while gesturing parallel to the volume axis. Skye responded,

Skye: "Uh . . . oh, you're right. You're totally right. So, [*gesturing along surface*] S is constant."

When the Green group responded to the first *Explore* prompt, they mention that other variables are held constant without specifying which other variables:

Laurel: "I think it is safe to say that, as volume increases, internal energy decreases."

Hunter: "Which makes sense . . . everything else is constant."

Later (well after considering the second *Explore* prompt), Laurel clarifies that S is held fixed along the path they chose when speaking with an instructor.

The Red group does not initially discuss the fact that they are implicitly holding S fixed, but Phoenix later repeats some reasoning about the first *Explore* prompt to Blaze, saying:

Phoenix: "So right here, is a point with a given entropy, and a given volume [*gesturing on surface*]. If I . . . keep the entropy constant and change the volume by moving up and down this [*gesturing*], along the surface and that's at different heights."

The above excerpts suggest that all three groups recognize that S is constant along the path they chose (parallel to the V axis) in order to answer the first *Explore* prompt. However, we did not see any evidence that students *chose* this path because S is constant along it; instead, they recognized that it is the case after the fact.

C. Realizing that S can change when V changes

In response to the second *Explore* prompt, the students in both the Green and Blue groups said that it is possible to change V without changing U. In the Blue group, this led to a realization that S must be allowed to change.

Ocean: "You just have to go around that line [*gestures along surface*]."

Skye: "Oh, you're totally, yeah, yeah, right. So we can change the volume . . . but are we allowed to change S?"

Ocean: "It just says 'without' [*nods*]. Yeah."

Skye: "Alright. Just stay on the level curves [*draws curve on the purple surface.*]."

Similarly, in the Green group, the students responded by acknowledging that S can change:

Hunter: "[*reading*] Is it possible to change the volume without changing the internal energy?"

Laurel: "Yes! But, [*stops writing, grabs purple surface, and moves finger along the surface.*] wait . . . [*still moving finger along surface in different directions*] I mean, yes, as long as you change the . . . whatever S is."

Hunter: "Yeah, yeah, so as long as S changes accordingly with it, in conjunction with it . . . cause I mean obviously there's lines, there's levels of internal energy . . . . If you follow that line across, volume is changing but so is S, whatever S is; position or something . . . . So as long as they change together"

In the Red group, Valentine suggested moving along an energy contour (or possibly a temperature contour). Phoenix replied by saying,

Phoenix: "So that'd be; so if you're going along the curves on here [*moves finger along purple surface.*], that would be a change in both volume and entropy, and internal energy. Because this is not the same height as you go along."

Phoenix clearly recognizes that S, V, and U can all change along the same path, but the Red group does not come to a consensus in response to the second *Explore* prompt.

All three groups thus recognize that it is possible to change S and V simultaneously, but no group connects this fact to their answer to the first *Explore* prompt (all three groups stated that U decreases when V increases).

D. Choosing a surface by matching the axis variables

The students identified the purple surface as having information that can be interpreted more directly than the blue surface or either of the contour maps. Both the Green and

the Blue groups initially considered the blue surface, then rejected it for answering the first *Explore* prompt. This is consistent with the instructors' observation that almost all (if not all) groups in the class used the purple surface to answer the *Explore* prompts.

In the Green group, while starting to respond to the first *Explore* prompt, Laurel holds the blue surface and suggests,

Laurel: "So, I think we should look at this one. This one's temperature and pressure."

But after both students paused and reflected for a few moments, Laurel exclaimed "Oh!" and switched to the purple surface. When the students reached the *Representation* prompt, they affirmed this selection of the purple surface.

Laurel: "Yeah, I mean we could think like, well as volume goes up, pressure goes down."

Hunter: "Yeah, so you could relate it"

Laurel: "So you could relate it but it seems like extra, just an extra step that doesn't really"

Hunter: "Yeah."

Thus these two students consider using the blue surface but decide that it seems like an "extra step" to use the blue surface by relating an increase in V to T and p.

In the Blue group, Skye starts responding to the first *Explore* prompt using the blue surface, but says " Oh, wait." Then Ocean taps on the purple surface while correcting, "should be on this one." Skye agrees that, "Yeah, that makes more sense." and the group continues with the purple surface. When responding to the *Representation* prompt, Skye said:

Skye: "Right. We chose the one with the two variables that were asked about."

Skye later expressed a belief that knowing T and p should be enough to define a state completely:

Skye: "Well, besides the ideal gas stuff, I feel like we should... there are two states, uh [*touching T and p on the blue surface*] another two state variables, so that's enough to define everything, but... [*Ocean says something unclear while pointing to worksheet.*] But not to answer the question, yeah! Good point. Okay."

So, despite some understanding of state variables, these students ultimately judge the blue surface to be unhelpful for responding to the *Explore* prompts.

The Red group did not initially try to use the blue surface, but Phoenix and Valentine discussed which surface to use at the end of the first *Explore* prompt:

Valentine: "So here is star, red star. So we use this surface [*points to purple surface*] or that surface? [*points to blue surface*]"

Phoenix: "Uh, this surface [*points to purple surface*] 'cause this surface is um [*unclear*]."

Valentine: "I see that is [*points to blue surface*] uh, p and... um, that is p and what?"

Phoenix: "[*Picks up blue surface and holds it up*] p and temperature."

Valentine: "Oh, okay. Alright [*Valentine & Phoenix continue writing on worksheet*]."

Thus, the knowledge that the blue and purple surfaces represent $U(T, p)$ and $U(S, V)$, respectively, led all three students to identify the purple surface as more helpful for answering the *Explore* prompts.

IV. SUMMARY & DISCUSSION

When responding to the first *Explore* prompt, all groups chose a path parallel to the V axis. We have no evidence to suggest that any of the students considered alternative paths for increasing V at this point in the activity. However, several students seemed aware that S is fixed along that path. We believe that the students unconsciously inferred a constraint (that S was constant) in order to be able to provide a single response to the prompt. This is consistent with our analysis of pretests that this population of students took before the activity, which found, "students are 'stuck' on the idea that one axis variable must be held constant when reasoning about changes in variables in thermodynamics" [11].

The second *Explore* prompt triggered a discussion in all three groups about possibly changing S, but did not cause the groups to reconsider their responses to the first *Explore* prompt. This may be because the students were in "answer making" mode [13]; that is, they may have been focused on answering the next question on the worksheet rather than reflecting on whether or not their answers to the different questions were consistent. However, each group was later asked by an instructor to reflect on their responses in a portion of the videos that we have not yet fully analyzed. Our preliminary assessment is that these interactions drew the students' attention to their implicit choice, and to the reality that there is in fact a choice to be made.

The students all primarily chose the purple surface to answer the *Explore* prompts. Given the information on each graph, the purple surface is the best choice for the *Explore* prompts. Thus, the students appear to understand at least some of the features of the various graphs. We did not see the students evaluate features of the four graphs, but the students did use the best graph for answering the prompts. More research should be done to study how students interpret these graphs and how the students connect between the surfaces and the contour maps.

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