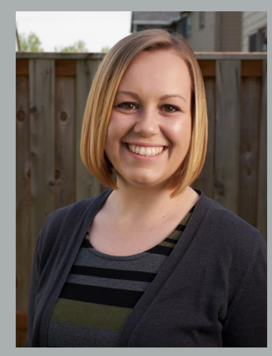
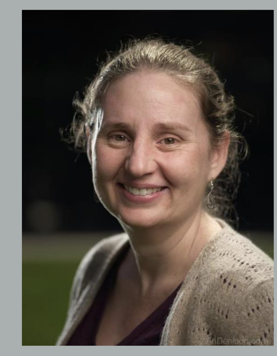
Student Application of Special-Case Analysis for Physics Sense-Making

Kelby T. Hahn, Paul J. Emigh, MacKenzie Lenz, and Elizabeth Gire





Research Questions

What do student special-case analyses consist of? Are there differences in student work between prompted and unprompted special-case analyses? If so, what are these differences?

Techniques of Theoretical Mechanics

Course Structure:

- 10 weeks, 50 minute meetings, 3 times per week
- Newtonian, Lagrangian, and Hamiltonian mechanics
- Special relativity
- Spring 2017 - Taught by author EG

Data Collected:

- Homework collected from 29 students
- 6 prompted special-case analysis analyzed

Sense-Making

Sense-making while solving physics problems involves coordinating the use of algebraic symbols with conceptual understandings, understandings of geometric relationships, and intuitions about the physical world.

- Treated on equal footing with the physics content goals
- Included on syllabus, exams, in-class discussions, and homework
  - Prescribed sense-making prompts on Homework 1-3, 9, & 10
  - Open-ended sense-making prompts on Homework 4-8

Special-Case Analysis

Special-case analysis involves setting parameters in a physics situation to special values to check if the result aligns with known results or one's physical intuition. See Special-Case Analysis Steps for more information.

- Prescribed special-case analysis prompts on Homework 2 & 6
- Special-case analysis was a common strategy employed on Homework 4-7

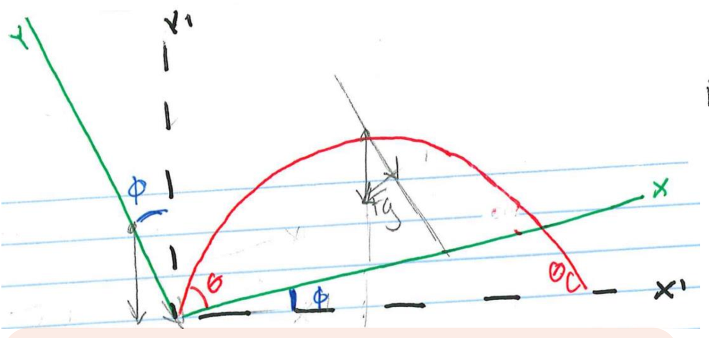
Prescribed Special-Case Analysis Prompt Example

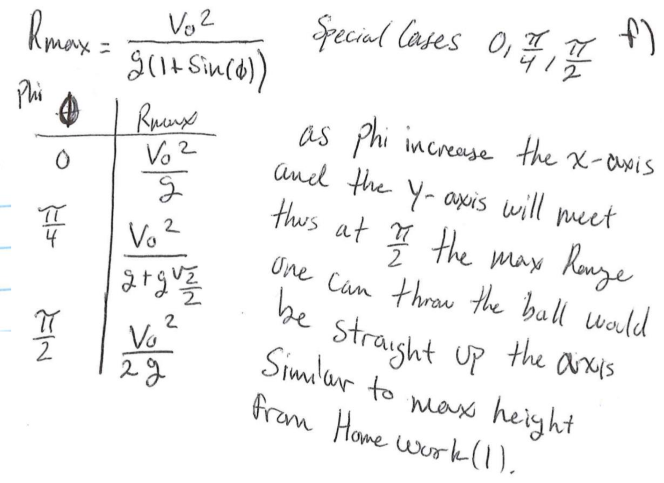
Question Prompt:

Projectile Over an Incline (*modified from Taylor 1.39*) A ball is thrown into the air, but the ground below is not flat - it makes an angle ϕ above the horizontal. The ball initially moves with speed is v_0 at an angle θ above the ground. Choose axes with x measured up the slope of the ground, y normal to the ground, and z across the across the ground (parallel to the horizontal).

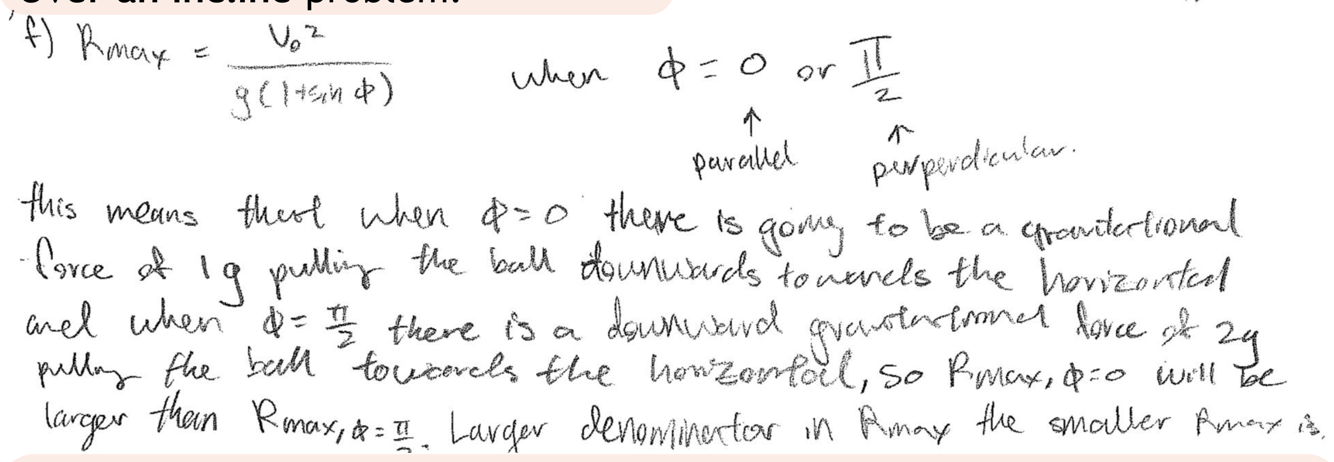
$$R_{max} = \nu_0^2/[g(1+\sin\phi)]$$

(f) *Sense-Making: Consider Special Cases* Does your result for the maximum range make sense for if the ground is horizontal? If the ground is vertical (like right up against a cliff)?



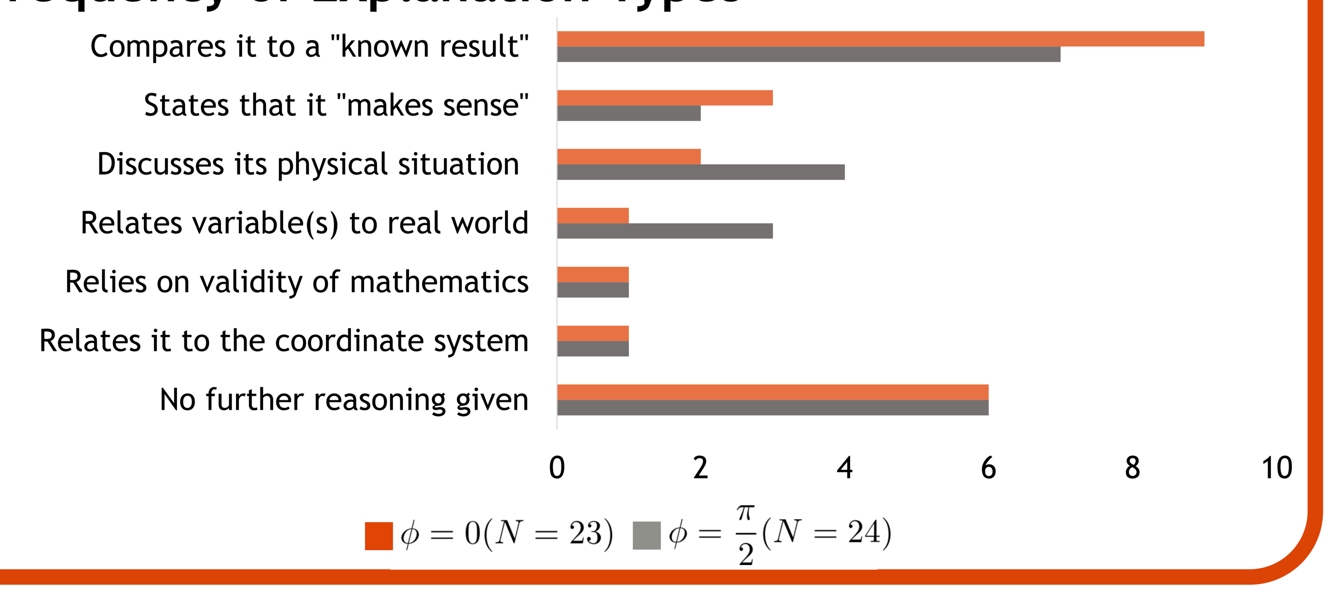


A student's drawing [above] of the physical situation of the **Projectile Over an Incline** problem.



Two examples of students' work [above] for a special-case analysis where φ = 0, π/4, & π/2. One student [upper right] compares their answer to a "known result," while another student [bottom] justifies their answer by discussing its physical situation.

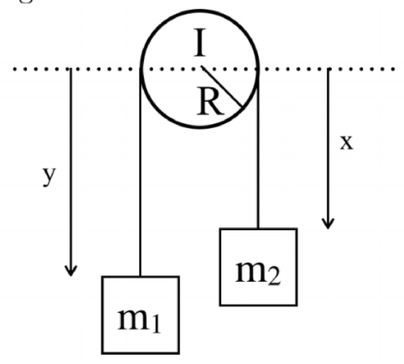
Frequency of Explanation Types



Open-Ended Prompt: Special-Case Analysis Example

Question Prompt:

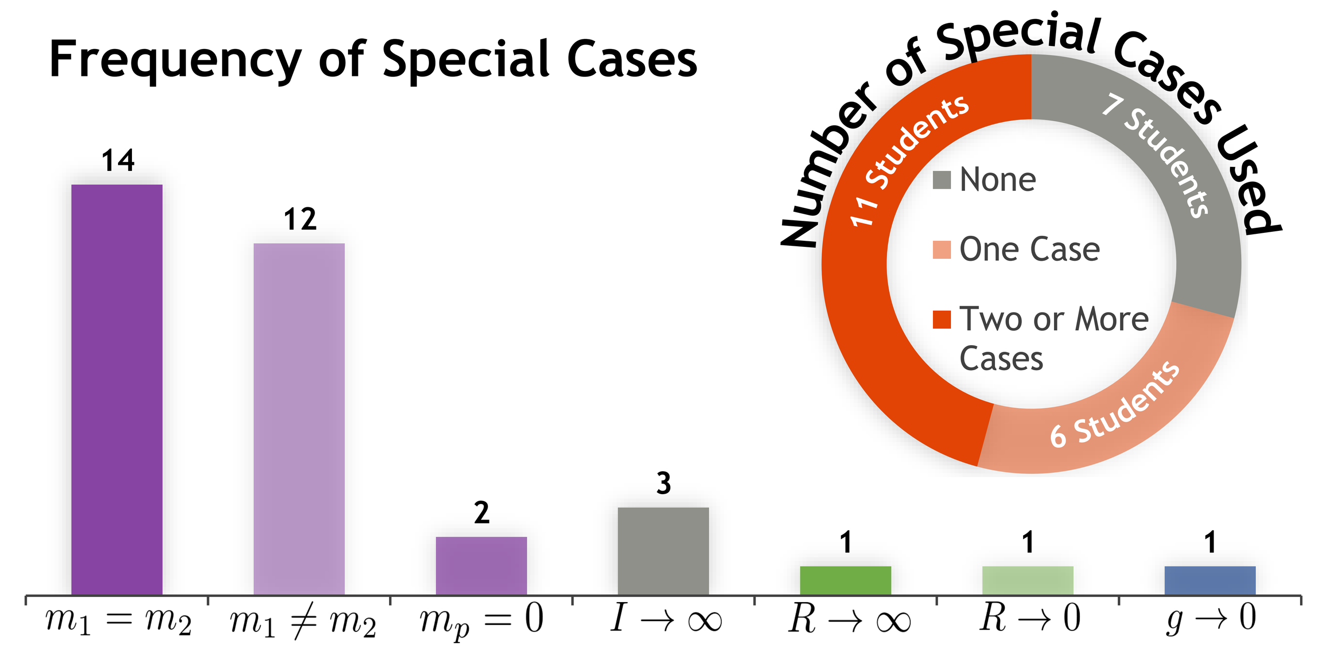
Atwood Machine Consider an Atwood machine, in which two blocks m_1 and m_2 are suspended by an inextensible string (length l) which passes over a pulley with moment of inertia I and radius R. The pulley has frictionless bearings.



(b) Write down the Lagrangian of the system and use it to find the acceleration of the system. *Sense-Making* Discuss at least 3 additional sense-making strategies for checking/interpreting the equation for the acceleration.

Students' responses to the open-ended prompt were coded and 33 special cases were found, performed by 18 different students. The cases that students chose to do were identified. Then students' further reasoning was coded similarly to the prescribed special-case analysis problem.

Frequency of Special Cases



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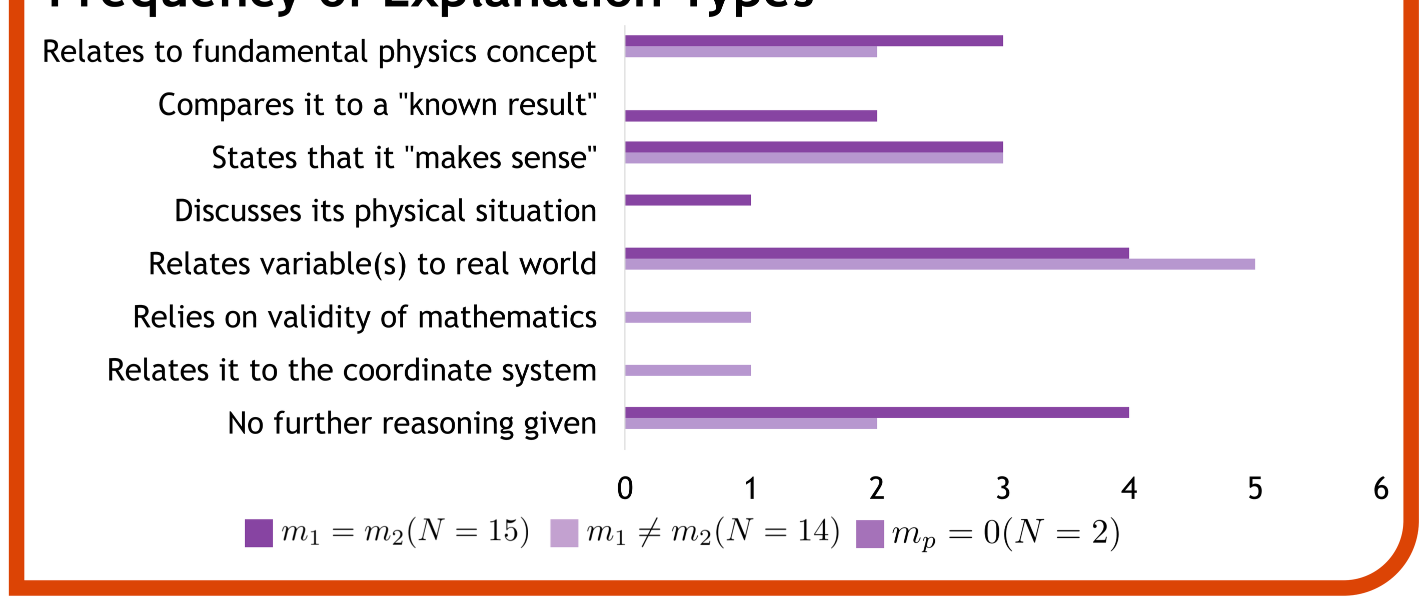

Special-Case Analysis Steps

Identify Case Conceptually/ Geometrically → Identify Relevant Value of Parameter → Evaluate Equation with Value of Parameter → Identify Expected Result → Compare Expected Result to Evaluated Result → Accept or Reject Equation as Answer to the Problem

Many of these steps can be done in different orders, above is the typical practice of students, though ideally we'd like to see them "Identify Expected Result" second.

Conclusions

Prescribed Special-Case Analysis

- Students can identify the case and the relevant value of the parameter
- Students tend to skip the "Identify Expected Result" step
- Students can evaluate the equation with the value of the parameter
- Students use a variety of techniques to analyze the result
  - Often because they skipped the "Identify Expected Result"
- Most common explanation type is comparing to a "known result"

Open-Ended Prompt: Special-Case Analysis

- Students chose to do special-case analysis often and on a large variety of problem types
- Student's didn't do special-case analysis on Hamiltonian of Free Particle or Crude Yo-Yo problems
  - Less common on problems with less strict parameters (ie. a rocket in free space)
- Students tended to do the same cases but often did not justify their choice of case
- Students used many of the same techniques to analyze the result as when prescribed
- Most common explanation type is "relating variable(s) to the real world"

Oregon State University

Acknowledgements
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References
J. R. Taylor, *Classical Mechanics* (University Science Books, 2005).
T. Dray, *Geometry of Special Relativity* (A. K. Peters, 2012).
A. R. Warren, Phys. Rev. ST Phys. Educ. Res. **6**, (2010).
B. Rosenshine and C. Meister, Ed. Lead. (1992).
M. Lenz and E. Gire, in PERC Proceedings (2016).
K. Hahn, P. Emigh, M. Lenz, and E. Gire, in PERC Proceedings (2017).
M. Lenz, K. Hahn, P. Emigh, and E. Gire, in PERC Proceedings (2017)