Teaching Gravitational Potential Energy: Student Interaction with Surface Manipulatives

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3D plastic surface models have been used to help students explore mathematical concepts related to functions of two variables. In order to understand how students reason with these models and their associated contour maps, we present analysis of three small groups of students working on an instructional activity designed to help students explore the relationship between gravitational potential energy and force for an Earth-satellite system. Students most often used the surface to examine functional behavior, discuss derivatives, and compare characteristics of the surface. Students often referred to the surface model in their explanations by pointing, gesturing, and tracing (without marking). Students held and drew on the surface less often, and rarely used the contour map. These results indicate that students may need additional support in using the contour map and encouragement to use props to visualize gradient vectors and viewing the surface from different vantage points.

I. INTRODUCTION

Energy is an important topic in physics, and it is used in virtually every area of physics, including mechanics, thermodynamics, and quantum mechanics. Beginning in introductory physics, students are taught to use energy concepts to solve problems. In particular, students need to understand and manipulate *potential energy* when solving a problem using energy conservation. Previous research on student understanding of potential energy has found that students struggle with certain aspects of potential energy, such as reading potential energy graphs, understanding that potential energy can be negative, and relating potential energy to force [1–3]

We designed an instructional activity to help introductory physics students understand gravitational potential energy (GPE). We chose a physical context of an object that is not necessarily close to the surface of the Earth. The students were provided with a 3D plastic surface model of the GPE of an Earth-satellite system (Fig. 1) and a matching contour graph of GPE. The activity asked students to reason about GPE by using the surface to graph GPE and gravitational force and by connecting GPE and force in both symbolic and graphical ways.

Our aim is to understand how the provided surface might promote student learning about GPE. Our specific research questions are:

1. "How do students interact with the surface?"
2. "What are the students trying to accomplish with each interaction?"

The answers to these questions will provide more detailed information about how and why students interact with the surfaces, which in turn will assist our larger research project of designing instructional activities with the surfaces.

II. PRIOR RESEARCH

Previous research on student understanding of potential energy, including gravitational potential energy, has tended to focus on identifying prominent student difficulties. For example, students often treat the minimum value of potential energy as zero and commonly believe that potential energy cannot be negative [1, 3]. Students also frequently misuse mathematical expressions for potential energy (such as $U = mgh$ instead of $U = -\frac{GMm}{r}$) and relate $\frac{dU}{dr}$ to force incorrectly [1, 3]. Students also tend to use the *magnitude* of the force to reason about the magnitude of the GPE; the stronger the force, the larger the GPE. However, the *derivative* of GPE is directly related to the force; the larger the derivative, the stronger the force. In one study, a student said that "since $PE = -\frac{GMm}{r}$, as you increase r, the total potential energy of the system will decrease" [2]. However, even though GPE does approach zero as r increases, GPE is negative, so it increases, rather than decreases, to zero.

Representations are likenesses or simulations of ideas, concepts, or objects. Internal representations exist only in the mind of the learner (memories, expectations, mental models, *etc.*) [4, 5], while external representations exist in the environment, (maps, graphs, equations, *etc.*) [6–9]. Manipulatives, like the plastic surfaces used in this study, are a type of external representation [10]. The contour map provided to the students and the graphs that the students are asked to draw during the instructional activity are also external representations. Learning with multiple external representations, both provided and student-created, "support different cognitive processes..., constrain interpretation options, and promote deep level learning" [11].

FIG. 1. A 3-dimensional plastic surface that is representative of the gravitational potential energy of an Earth-satellite system. The white circle shows where the Earth is located. There are three marked points on the surface (red star, green square, blue circle).

III. METHODS

This study is part of a larger project aimed at designing instructional material to help students reason about the multivariable functions encountered in physics classes [12]. In particular, our instructional activities make use of three-dimensional graphs ("surfaces") that represent physical systems. Activities that use surfaces have previously been used to help students learn multivariable calculus concepts [13]. This aspect of the project focuses on an activity that makes use of a surface representing the gravitational potential energy for an Earth-satellite system (see Fig. 1).

The activity (see Fig. 2) is intended primarily to help students reason about GPE, and gravitational force, and connect GPE to force. The activity was given to students in a recitation section during the second term of calculus-based intro-

Gravitational Potential Energy and Gravitational Force

Your group has a plastic surface and a contour map that represent the gravitational potential energy of a space station-Earth system as a function of the position of the space station relative to Earth. The gravitational potential energy is zero infinitely far away from Earth. Solve the following problems together and discuss the results.

You are employed by a company called SpaceY. SpaceY wants to put a space station at the blue circle.

(1) If the space station moves further away from Earth, how will the gravitational potential energy change? What if the space station moves closer to Earth?

(2) Identify other points on the surface where the gravitational potential energy is the same as the potential energy at the blue circle and draw a line to connect them. Do the same for the orange star and the green square.
 (a) Align your surface with the contour map. How are you making your alignment?
 (b) How could the space station move so that the gravitational potential energy remains constant?

(3) Sketch a graph of the gravitational potential energy (U) vs. distance from the center of the Earth (r). Remember to label your axes.
 (a) Does your graph match your answers to (1)? If not, reconcile any differences.
 (b) Why is it reasonable to represent the information from the surface in a graph with only 2 axes?

(4) What direction will the gravitational force at the blue circle be? Indicate the direction of the force at this point on the contour map.

(5) Locate a point where you would expect the gravitational force to be larger than at the blue circle.

(6) Is the rate of change of gravitational potential energy with respect to $\boldsymbol{r}$ positive, negative, or zero? Compare $\frac{dU}{dr}$ at the two points from (4) and (5). Which one has a larger magnitude?

(7) Sketch a graph of the gravitational force vs. distance from the center of the Earth. Remember to label your axes.

(8) The SpaceY employee handbook states:

> There is a relationship between gravitational potential energy and force; force is the negative gradient of gravitational potential energy. $F = -\nabla U(r)$

Do you agree? Support your answer with evidence from this activity. Recall that the gradient of a function, $\nabla U(r)$, is equal to $\frac{\partial U}{\partial r}\hat{r}$ as long the function depends only on r.

FIG. 2. The instructional activity prompts. The worksheet given to the students was two pages long and included space between the prompts for students to answer.

ductory physics, the same week that universal gravitation was covered in class. The recitation section was led by an experienced graduate student TA who prepared for the activity by working through the questions with author PJE (who was an instructor for the course). It is unlikely that any of the students had previously worked with a surface before, and the students who participated in the study did not necessarily have the same in-class preparation on the topic of gravitational potential energy.

We recorded video and audio of three groups of students working on the activity, then transcribed the audio and the students' interactions with the surface. We then performed a Thematic Analysis on the aspects of the videos related to the surfaces [15]. This analysis was guided by the categories of interaction from Activity Theory [14], which are split into three levels of increasing intentionality: operations, actions, and activity (see Fig. 3). Concisely, operations are how something is done, the actions are what is done, and the activity is why it is done [14]. Operations are the basic tasks that students do: write an equation, draw a picture, move the surface, *etc*. Actions are more goal-oriented, intentional things that students do: describing functional dependence, depicting a scene, showing something to a group member. The highest level of goal-oriented, intentional student activity in this case is accomplishing the instructional task.

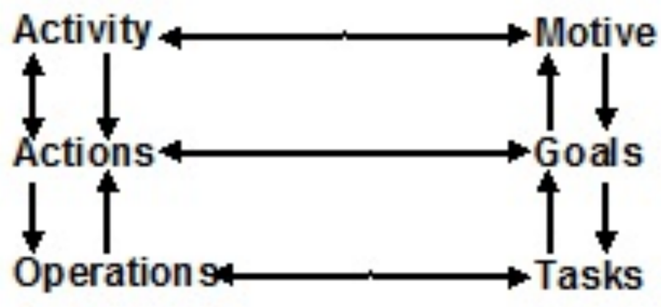

FIG. 3. Hierarchy of Activity Theory. The vertical arrows represent possible movement between categories and the horizontal arrows connect activity theory terms with their corresponding meanings. (Figure reproduced from Ref. [14].)

The first step in our analysis was to identify each interaction/operation with the surface. Then, we examined the transcript surrounding each operation more closely to determine the intentional actions associated with each operation. Since this paper is focused only on the students' interaction with the surface, we did not associate an action with any non-surface operations; however, some individual operations corresponded to more than one action.

IV. RESULTS

In this section, we first introduce the operations students performed and then describe the actions they performed. We then briefly discuss the meaning of each operation and action category as well as some initial patterns in how the actions and operations correspond.

A. Operations

The students interacted with the surface in five key ways, which are described in Table I. Most of the operations were relatively straightforward to identify. Students most com-

TABLE I. Operations students performed with the surface and the number of instances that operation was identified across the groups.

Operation	Description	N
Tracing	Students used finger or other object to trace self-drawn or imaginary paths on the surface.	41
Pointing	Students used a finger or other object to point at features of the surface.	43
Holding	Students lifted, turned, or moved the surface as well as moved themselves around the surface.	12
Gesturing	Students used hands or fingers to motion at larger features of the surface.	35
Drawing	Students were given dry-erase markers to write and draw on the surface.	18

FIG. 4. A student using a marker to depict a tangent on the surface.

monly traced a path across the surface, pointed at a feature of the surface, and gestured at the surface. The Pointing category was characterized by motioning at small, specific features of the surface (like a marked point), while the Gesturing category was characterized by motioning at larger, more general features of the surface (like the side of the surface). One particularly interesting operation in the Gesturing category was that one of the three groups placed a marker on the surface to depict a tangent line (in the radial direction) to represent the value of the gravitational force (see Fig. 4).

Students also occasionally drew on the surface with dry-erase markers (students were told that the surfaces are dry-erasable at the start of the activity). Lastly, the students sometimes picked up the surface to align it with the contour map or examine the surface from a different angle. One notable Holding operation was when each group of students looked at the surface in profile (such a profile is highlighted in yellow in Fig. 5) while drawing a graph of GPE *vs.* distance.

B. Actions

Students interacted with the surface for several reasons, the most common of which are described in Table II. Most commonly, students talked about how GPE increases with increasing distance, and they would try to determine how quickly GPE or gravitational force increased. The Examining Functional Behavior category was most prevalent when students were drawing graphs of energy (Q3) or force (Q7). Students often traced the surface while discussing functional behavior to mimic the surface increasing or decreasing along a specific path.

Students compared characteristics most often when discussing prompts 5 and 6 (Fig. 2), which asked students to compare the force and the derivative of GPE at two different locations. While determining where the force would be larger than it is at the blue circle (Q5), one student gestured towards the Earth on the surface and suggested that anywhere closer to the Earth than the blue circle would have a larger force. The student reasoned that "there's way more gravity on Earth than in outer space" and motioned from the Earth outward.

Students often discussed slope, derivative, or rate of change, most often when answering prompts 5-8 (Fig. 2). This category does not include discussions about force without mentioning derivative, rate of change, or slope. This category was most often associated with the Pointing and Gesturing operations. We also noticed that one group of students (as discussed above) accounted for most of the actions in this category when they indicated possible tangent lines (as in Fig. 4).

Students also used the surface to discuss specific variables, such as the direction of the radial coordinate (r). Another example of this was when students pointed out that the height of the surface represents the GPE. This category also includes actions where students determined the sign of the gravitational force by comparing the direction of the force vector to the direction of $\hat{r}$.

Some of the students' actions were specifically prompted by the activity. For example, one prompt asked students to "locate a point where you expect the gravitational force to be larger than at [a previous point]." When a student pointed at or drew a mark at that location, that operation was categorized as Prompt. In the activity, we asked students to draw on the surface, align the surface with the contour map, and draw on the contour map. Drawing on the surface accounted for 16 out of 23 actions in the Prompt category, while aligning the surface (Contour Map and Holding in operations) accounted for the other 7. Students did not draw on the contour map. We note that almost all actions were taken in response to prompts from the worksheet, but that the Prompt category only includes those actions which were directly prompted.

The less common categories (not shown in Table II) were

TABLE II. Actions related to the surface taken by the students and the number of instances that action was identified across the groups.

Action	Description	N
Examining Functional Behavior	Students examined how GPE or gravitational force change with distance.	45
Comparing Characteristics	Students compared characteristics of two or more places on the surface.	31
Examining Slope/Derivative	Students explicitly discussed the derivative, rate of change, or slope.	29
Following a Prompt	Students performed these operations because the instructional activity explicitly told them to.	23
Discussing Characteristics of a Variable	Students discussed the characteristics of a specific variable, for example, the direction of r.	17

FIG. 5. The surface used in the activity. One side is highlighted, demonstrating what is called the "profile" in this paper.

Determining Location, Just for Fun, Unclear Motive, and Other. Operations in the Location category were used to discuss the location of objects, for example, confirming the location of the Earth on the surface. The Fun category includes actions that served no instructional purpose and made at least one student smile or laugh. This includes a student drawing a "planet" on the surface and showing the group and a student repeatedly placing a marker on the surface and catching it as it rolled off. Contrary to the Other category, which contains operations that had a discernible purpose that did not fit in another category, the Unclear category contains operations that did not have a discernible purpose.

V. DISCUSSION AND IMPLICATIONS

As seen in Table. I, most interactions with the surface were Tracing, Pointing, and Gesturing, showing that the students did not take advantage of certain features of the surface: Drawing and Holding. Of the 18 times students drew on the surface, 16 were explicitly prompted by the activity. The other two drawings were a "planet" drawn for fun and a mark drawn and immediately erased. Drawing on the surface was introduced early in the activity; Prompt 2 (Fig. 2) instructs students to connect all points of equal GPE for each of the marked points on the surface. The TA also announced that the surfaces are dry-erasable at the beginning of the activity. However, students rarely drew on the surface without being explicitly prompted to do so. One explanation for this may be that the students were inexperienced with the surfaces and their features. Furthermore, the students may not have felt a need to *draw* on the surface—they traced paths on the surface much more commonly—which may be an indication that this version of the activity does not currently leverage all of the features of the surface.

Students only interacted with the contour map when instructed by the activity to align the surface with the contour map. A later question (Q4) asks students to draw a force vector on the contour map—yet every observed group drew the force vector on the surface instead. There are several possible explanations for this: the students had already placed the surface on top of the contour map and may have been averse to disrupting the alignment, the students were inexperienced with contour maps, and there was little information on the contour map. The contour map was purposefully designed with no words, units, or values so that students would not be distracted with computing numbers rather than examining qualitative behavior. However, the lack of numbers or labels may have deterred the students from using the contour map since it did not contain any information not also present in the surface.

As seen in Table II, the students used the surface most often to examine functional behavior, compare characteristics at different locations, and discuss the derivative, the slope or the rate of change. These actions all involve offloading cognitive function to the surface. Representations like the surface allow students to visualize variables and their relationships and to manipulate a system that would otherwise be difficult or impossible to manipulate [16]. Examining functional behavior, comparing characteristics, and discussing the derivative, the slope, or rate of change without a surface or graph would require more work from the students' internal representations; the surface and graphs allow students to put some of that work onto external representations. This makes it easier to discover and communicate ideas since the students no longer need to visualize and manipulate the system in their heads, and each student can see the same representation.

Each group of students interacted with the surface between 50 and 70 times during the 50 minute instructional activity. Out of the 173 total interactions, only 23 were explicitly prompted by the activity. This implies that most of the interactions were done because the students found the surface useful in explaining their thinking. Students were able to use the surface to offload cognitive function and discuss abstract concepts, like functional behavior and derivative, that may have been too difficult otherwise.

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