Sensemaking in special relativity: developing new intuitions

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Special relativity is both exciting and challenging in that it requires developing new intuitions about relativistic situations. How can we help students make sense of special relativity when their intuitions are classical? This paper will discuss student sensemaking about special relativity in a sophomore-level course designed to explicitly teach and support physics sensemaking. The course particularly emphasizes two sensemaking strategies: visualization with spacetime diagrams and the development of *rules of thumb*. Rules of thumb, like "proper time is the shortest time," serve as footholds when solving problems in special relativity. Specifically, we present an analysis of students' use of *rules of thumb* in their written solutions to homework problems. We found that students draw upon time rules, length rules, and relativity rules to solve the Twin Paradox. We also discuss how *rules of thumb* fit with other theoretical constructs.

I. INTRODUCTION

Physics sensemaking has been established as an important expert-like activity for students to develop [1–5]. Sensemaking is commonly conceptualized as drawing on intuitions and developing conceptual understandings [5–14]. The topic of special relativity presents unique challenges for student sensemaking because physics students' intuitions are primarily classical and often contradict relativistic results [15–18].

Students ideas about the non-classical results of special relativity are often disconnected [19]. These unconnected understandings make researching students' relativity knowledge challenging [20]. For example, Scherr [21] reported that students often concurrently hold contradictory ideas about simultaneity. A study by Cormier and Steinberg [22] notes that students' epistemological differences may contribute to their ability to reconcile relativistic ideas with their other physics understandings. The researchers discuss a student with a sensemaking-focused epistemology who often uses short expressions of information, like "no frame is absolute," to aid them in making sense of the relativistic problems. The expressions Cormier and Steinberg discuss align with what we call *rules of thumb*. Drawing on *rules of thumb* can be thought of as a sensemaking or problem-solving heuristic: a strategy or approach that does not guarantee a successful outcome [23, 24].

In this paper we explore the following research questions:

RQ1 What *rules of thumb* do students invoke for solving the Twin Paradox?

RQ2 What role do *rules of thumb* play in student reasoning about special relativity content?

II. INSTRUCTIONAL CONTEXT

Techniques of Theoretical Mechanics is a course at Oregon State University, a large, public, research-intensive institution, designed and taught by author EG. The course is meant to ease the transition between introductory and upper-division physics through explicit instruction in physics sensemaking strategies. This course included 6 weeks of classical mechanics and 4 weeks of special relativity content, including time dilation, length contraction, Lorentz transformations, spacetime diagrams, hyperbolic geometry/trigonometry, relativistic velocity addition, and conservation of four-momentum. The class met for 50 minutes 3 times per week and small-group problem-solving activities happened at least once per week.

The prerequisite courses were multivariable calculus and the first 2 quarters of introductory physics (which include Newtonian mechanics for translational and rotational motion and waves). Of the 50 students who took the final exam, a subset were enrolled in other noteworthy courses. Nine students were co-enrolled in the last quarter of the introductory physics sequence, 9 students had completed the introductory sequence the previous quarter, 2 students had not yet taken the last quarter of the introductory physics sequence, 20 students had already taken some of the junior-year *Paradigms* courses [25], and 5 students were co-enrolled in a general relativity course.

Techniques of Theoretical Mechanics is unique in that it includes an explicit emphasis on physics sensemaking. Sensemaking was emphasized as heavily as the physics content of the course: it appeared on the syllabus, was prompted in homework problems and exams, and was discussed in nearly every class meeting and office hours. The instructional team (EG, KTH, and two undergraduate learning assistants) encouraged students to engage in sensemaking at all stages of the problem-solving process. During class discussions, both within their groups and as a whole class, students were expected to engage in sensemaking. This expectation included being able to discuss the correctness of their answers and to articulate their conceptual understanding of the problem.

To aid students in sensemaking about special relativity, the content was taught with an emphasis on building new intuitions for relativistic scenarios. Lectures often included skits where the instructional team acted out the reference frames, which were then explicitly discussed alongside traditional rules of thumb like "moving clocks run slow" and "proper length is the longest length." Following Dray's [26] textbook, hyperbolic geometry and spacetime diagrams were used to discuss relativistic situations. Students were also encouraged to draw on sensemaking skills they had developed in classical mechanics for the relativistic situations.

Homework was assigned each week with the first 8 assignments focused on classical mechanics and the last 2 on special relativity. The course, as a whole, was structured to support students' development of sensemaking skills through use of Rosenshine's [27] scaffolding-and-fading approach. Early prompts guided students to do specific sensemaking strategies with specified physical quantities. By the time students were working on special relativity content the sensemaking prompts were faded to: "For each problem, use several strategies to make sense of your answers to each problem." For relevant problems, spacetime diagrams were prompted in addition to this global sensemaking prompt.

The majority of the students analyzed are male physics majors at a large, public, research institution. Due to the nature of homework assignments the written work analyzed may be a polished version of the students' reasoning and may not be representative of one individual's thought. Students are strongly encouraged to work together and discussions in the teaching team's office hours were primarily about homework questions. Additionally, as this paper focuses solely on analysis of written work, claims here are made from interpretations of what students articulated on their assignments. We do not claim they articulated the full range of their understandings.

III. METHODS

In this paper we describe a subset of the various rules of thumb that students invoked in their problem-solving of the canonical Twin Paradox, (see Fig. 1). Students were prompted to solve this problem on their final homework assignment during week 10 of the course. In addition to the physics context prompts, the assignment included a global prompt asking students to: "use several strategies to make sense of your answers to each problem." Homework assignments were scanned and all student work pertaining to the Twin Paradox problem was analyzed for instances of students invoking rules of thumb.

We used a phenomenographic approach [28] to identify the various rules of thumb students invoked and then performed a thematic analysis [29] to draw connections between the rules. Author KTH read through all assignments multiple times to identify any instances where students made an assertion about relativity concepts. Each of those instances was given a short descriptor that summarizes student understanding. Those descriptors were then combined into categories henceforth referred to as *rules of thumb*. Any instances where the student assertions were unclear or unique were discussed with PJE. Once the rules of thumb were articulated, themes among the rules were identified. KTH then went back through the students' instances of asserted understanding to analyze what role the rules of thumb seemed to play in the students' assertions. When analyzing student work, instances where students only referred to "length contraction" or "time dilation" with no further elaboration were not included as rules of thumb. This choice was made in part because these phrases were used primarily to justify which equation they are using rather than as assertions of more conceptual understanding.

IV. RULES OF THUMB

Students expressed a variety of ideas about both special relativity in general and the uniqueness of the Twin Paradox. The rules of thumb students invoked were of three types: time rules, length rules, and relativity rules. The rules of thumb served two main purposes: (1) to aid students in making calculations of lengths and times and (2) to assist in interpreting or justifying the results they calculated.

A. Time Rules

Time rules were the most common type of rules of thumb present in students' work. This is unsurprising because the Twin Paradox asks about the twins' ages (time) to outline the asymmetrical element of Nut switching reference frames. Students often drew on time rules to address the age discrepancies they had calculated for the twins.

The rule of thumb students most commonly invoked was *moving clocks run slow*. Students often used this rule to interpret how old Geb is in part (b), as both students quoted here do. This first example shows a student using the idea of *moving clocks run slow* to justify why Nut (the moving twin) should be younger:

> *"Nut was moving fast so therefore she should be younger ✓"* – Student 10

Comparably, this other student used the same idea of *moving clocks run slow* to justify why Geb (the twin on Earth) should be older:

> *"It makes sense that Geb would be older because he is the one who remained 'stationary' at home."* – Student 1

Similarly, it is unsurprising that *moving clocks run slow* is the most common rule of thumb to be invoked. Not only was it discussed in class, but this idea is the seemingly self-contradictory element of the Twin Paradox (each twin should see the other's clock moving slow). Surprisingly, students rarely addressed this paradoxical nature explicitly and did not seem to use this rule of thumb to aid in identifying the paradox.

The other way that students discussed time rules was with respect to proper time. We found three ways that students seemed to be thinking about proper time: as the shortest time, as the time between colocated events, and as moving time. Both *proper time is the shortest time* and *proper time is the time between colocated events* were rules of thumb discussed

Instructions: For each problem, use several strategies to make sense of your answers to each problem.
The Twin Paradox (modified from Griffiths 12.16) On their 21st birthday, one twin, Nut, gets on a moving sidewalk, which carries her out to star X at speed $4/5c$. Her twin brother, Geb, stays home. When Nut gets to star X, she immediately jumps onto the returning moving sidewalk and comes back to earth, again at speed $4/5c$. She arrives on her 39th birthday (as determined by *her* watch). (The names are based on twins in ancient Egyptian mythology.)

(a) Draw a single spacetime diagram showing the entire trip in the reference frame of Geb. Your diagram should show the world lines of both twins. Label all events. Update your spacetime diagram as you answer the following questions.
(b) How old is Geb (who stayed at home)?
(c) How far away is star X? (Give your answer in light years.)
Now, call the outbound sidewalk system S' and the inbound one S'' (the earth system is S). All three systems set their master clocks, and choose their origins, so that $x = x' = x'' = 0,\ ct = ct' = ct'' = 0$ at the moment of departure.
(d) What are the coordinates (ct, x) of the jump (from the outbound to inbound sidewalk) in S, the reference frame of Geb?
(e) What are the coordinates (ct', x') of Nut's jump in S'?
(f) What are the coordinates (ct'', x'') of the jump in S''?
(g) If Nut wanted her watch to agree with the clock in S'', how would she have to reset it immediately after the jump? If she *did* this, what would her watch read when she got home? (This wouldn't change her *age*, of course—she's still 39—it would just make her watch agree with the standard synchronization in S''.)
(h) If Nut is asked the question, "How old is Geb right now?", what is the correct reply:
  i. just before she makes the jump?
  ii. just after she makes the jump?
(i) How many earth years does the return trip take? Add this to (ii) from (h) to determine how old *Nut* expects Geb to be at their reunion.

FIG. 1. The Twin Paradox prompt given to students, modified from Griffiths 12.16 [30]. The global instruction asking students to "use several strategies" is an example of the fading in our application of Rosenshine's [27] scaffolding and fading approach.

in class and often are used to define proper time for students. *Proper time is moving time* was never explicitly stated in class and may show evidence of how students are grappling with defining proper time for themselves. It may also be a way students read about proper time being discussed in textbooks or internet resources.

Proper time is the shortest time was invoked both as a means of identifying which twin was experiencing proper time and as a means of justifying the amount of time a twin experienced. Similar to *moving clocks run slow*, students often invoked this rule of thumb in response to part (b) of the problem, as both examples do. This first quote is from a student who is using the fact that they calculated Nut's time to be shorter to claim Nut's time is proper.

> *"Because Nut measures the shorter time, she measures the proper time (τ) for this problem."* – Student 16

This approach is of particular interest due to the fact that one time is shorter than the other does not inherently mean that one is the proper time. In contrast, Student 14 uses some other, unspecified means of determining that Nut measures proper time to justify their conclusion that Geb's time is longer.

> *"This makes sense because Nut measures proper time, so Geb should measure a [longer time]"* – Student 14

The other rule of thumb students were told as a definition for proper time is *proper time is the time between colocated events*. Some students explicitly discussed the events that were colocated (Nut aging), like the quote below shows.

> *"Nut measures proper time because her age changes at herself, so she perceives the changes to be colocated."* – Student 1

Other students were less explicit about which events were colocated but still relied on location to determine who measured proper time.

> *"Nut stays in the same place in her ref frame. She experiences proper time."* – Student 33

As mentioned previously, colocation was emphasized in class as *the* way to determine proper time for two events. Despite this emphasis, students invoked *proper time is moving time* as a means of determining proper time. For example, in the quote below Student 27 is considering part (h) i.[P: ?] and concludes that Geb is measuring proper time because he is moving at $\frac{4}{5}$c. After this statement they go on to use this information to calculate what Geb's clock would measure for the time between the same two events.

> *"To Nut, she's been traveling 9 years, and Geb is moving $\frac{4}{5}$c, so Geb is holding propertime [sic]."* – Student 27

While this line of reasoning can often lead to correct conclusions about proper time, it breaks down if the events are not colocated in the "*moving*" frame.

These time rules that we have identified show that students have a variety of understandings (rules of thumb) about time dilation that they can flexibly apply as both orienting and evaluative sensemaking strategies.

B. Length Rules

Length rules were the least common in our dataset. This may be due to the fact that the original problem statement gives no explicit lengths or it may be related to the specific sub-prompts students were asked. Most instances, and all quotes here, refer, at least in part, to how far away star X is, which is prompted for in part (c) of the problem statement.

In many ways, the length rules parallel the time rules. Similar to *moving clocks run slow* students drew on the rule that *moving rulers are shorter*. In all instances, students drew on the fact that the frame was moving to conclude that length had contracted.

> *"However, this is a contracted distance. Star X is moving @ $\frac{4}{5}$c relative to Nut."* – Student 28

Again, similar to the time rules, students were given two definitions for proper length in class: (1) *proper length is the longest length* and (2) *proper length is the distance between simultaneous events*. Unlike the time rules, we did not see any instances of students' written work drawing on the definition of *proper length is the distance between simultaneous events*. That is not to say that students did not discuss simultaneity or proper length separately, but students did not seem to relate the two ideas.

Proper length is the longest length was only invoked by one student in once instance: when reflecting on the problem and comparing their answers to parts (c) and (f).

> *"Since the proper length is always the larger of the two lengths, Nut is the most prime candidate for the proper length."* – Student 23

Here they used the fact that they had calculated Nut's length as longer to determine that she measured the proper length. No further reasoning was expressed beyond identifying Nut's length as the proper length.

Rather than drawing on the rule of thumb definitions given in class, students more frequently drew on the rule of thumb that *proper length is at rest*. The student quoted below uses this idea to make sense of the distance star X is away, part (c).

> *"Geb measures the proper length to the star because he is at rest in the Earth-star reference frame."* – Student 1

As with the time rules, this line of reasoning can often lead to correct conclusions about proper length but it breaks down if the "*at rest*" frame the length between events is measured in is not the frame where the events are simultaneous.

Students drew much less often on the length rules to solve and interpret the Twin Paradox. Despite this, it is evident that the rules of thumb adopted about length parallel those for time in many ways.

C. Relativity Rules

We found three more rules of thumb related to students' understandings of special relativity as a whole: (1) *nothing* *can travel faster than the speed of light*, (2) *relativity requires unique intuitions*, and (3) *each observer sees the other as "moving"*.

The most common relativity rule is *nothing can travel faster than the speed of light*. This rule was invoked by many students to make sense of their results. For example, the quote below is of Student 30 interpreting their coordinates of the jump in the inbound (S") frame, part (f).

> *"Since light travels faster than all, it makes sense that ct'' is greater than x''."* – Student 30

Other students used this rule of thumb to aid them in contemplating the equations themselves. The fact *nothing can travel faster than the speed of light* is arguably one of the most well known rules of thumb of special relativity, so it is unsurprising that students draw on this familiar idea to make sense of their results.

Two students seemed to draw on a rule of thumb about the unique intuitions of relativity: *relativity requires unique intuitions*. Student 23 expresses a need for new guidelines for relativity when reflecting on their problem as a whole, when they conclude that special relativity has guidelines that their results coincide with.

> *"Making Sense of Nut's predictions: The paradox demonstrates that neither twin can agree on the time the other experiences, in other words, they seem to experience different time at different speeds, which agrees with special relativity principles."* – Student 23

The last rule of thumb we discuss here is that *each observer sees the other frame as "moving"*. This rule was only invoked by one student, but in many ways is the essence of the Twin Paradox and thus is important to highlight. During their final reflection on the problem this student concludes that each twin believes the other twin is experiencing proper time.

> *"Because Nut believed that Geb experienced proper time, Nut believed the journey for Geb was less time than Nut. However, Geb thought the exact thing towards Nut (lone behold [sic] Nut was measuring proper time for the journey), and thus each believed that the other was experiencing proper time during the trip."* – Student 4

This student is grappling with the paradoxical elements of the problem without expressing the resolution of the paradox.

V. DISCUSSION & IMPLICATIONS

Students drew on a large variety of rules of thumb when solving the Twin Paradox problem. These rules seemed to serve two purposes for students: (1) as a means of orienting to the problem and (2) as a means of reflecting on their answers. This is most obvious when comparing the two students quoted using the *proper time is the shortest time* rule of thumb. Student 16 uses the rule of thumb to reason about the situation and orient themselves to the lengths they expect, whereas Student 14 uses the rule to justify their answer. This pattern of the rules of thumb being used for both orienting and evaluative sensemaking persisted through many of the rules.

Rules of thumb also seem to play the role of memes: units of cultural transmission or imitation [31]. As Dawkins describes, memes are a "unit of convenience" for discussing cultural transmission of ideas. These ideas are replicated through imitation and permeate a culture. The special relativity rules of thumb identified here are the memes students found useful from the special relativity culture they experienced: lectures, in-class activities, homework, textbooks, online-resources, *etc*. Dawkins himself discusses the uptake of scientific memes: "If the meme is a scientific idea, its spread will depend on how acceptable it is to the population of individual scientists" [31]. Treating rules of thumb as memes is advantageous for discussing how students adopt, apply, and potentially propagate the rules. Memes do not have perfect copying-fidelity and are subject to continuous mutation and blending, just like we see here with the rules of thumb that students adopted and reinterpreted. This classification, of rules of thumb as memes, could provide deeper insight into the cultural role rules of thumb play in aiding students in developing expert-like reasoning.

While this study scratches the surface of student engagement with rules of thumb, we think these rules play an analogous role in special relativity to the role of phenomenological primitives (p-prims) in classical mechanics [32]. That is not to say that rules of thumb are p-prims— as they are not intuitions built on lived-experiences—but students use them as if they are self-explanatory and seem to be invoked as a whole. Once rules are accepted by students they provide the necessary foundation for explaining phenomena, as p-prims do. Unlike p-prims, the rules can be broken into smaller components and explained, even though students often treat them as if they do not need to be.

While this work shows parallels to p-prims, future work utilizing a p-prim lens is needed to begin to fully understand how the roles of rules of thumbs and p-prims relate to one another in relativistic contexts. Lastly, researchers seeking to understand how students develop intuitions about special relativity, and other non-classical physics content, need to consider not only the rules identified here but the way they are spread, adopted, and adapted by students. Thinking of the rules as memes provides a structure for analyzing those elements of student reasoning.

Instructors can introduce rules of thumbs to students as sensemaking tools in special relativity. Instructors need to be aware that, just like the low copying-fidelity of memes, students misapply and transform rules of thumb.

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