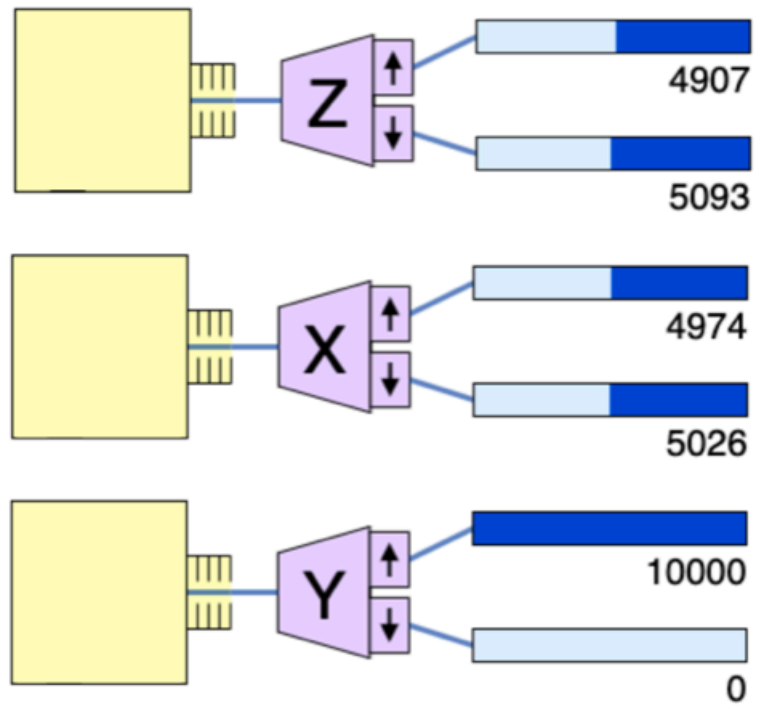
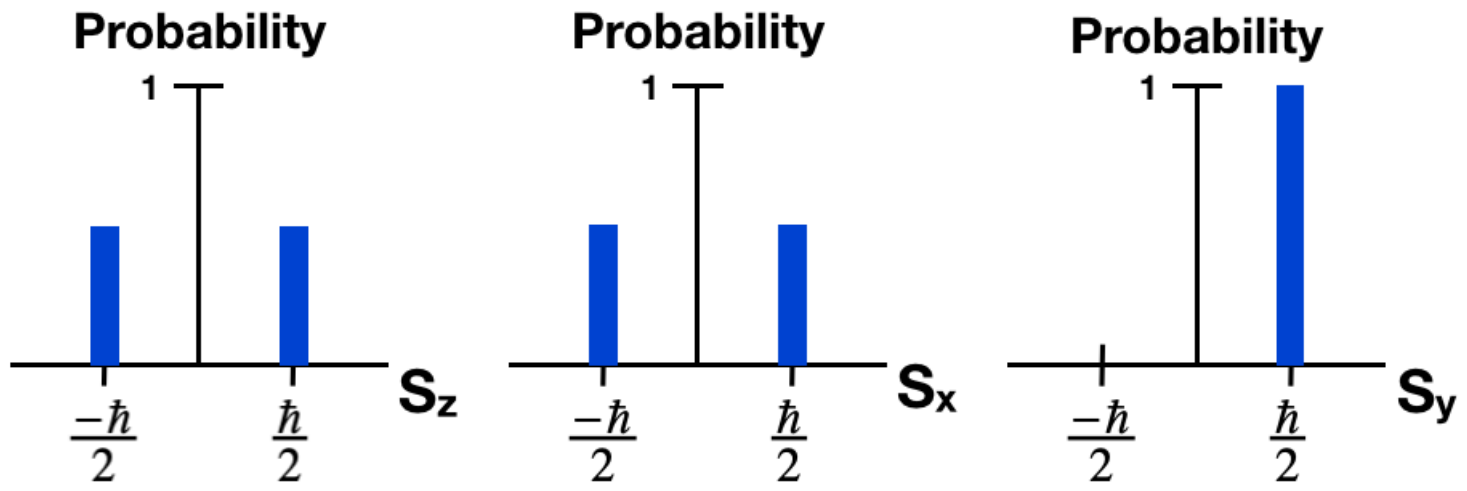
Making sense of quantum mechanics with the languages of physics

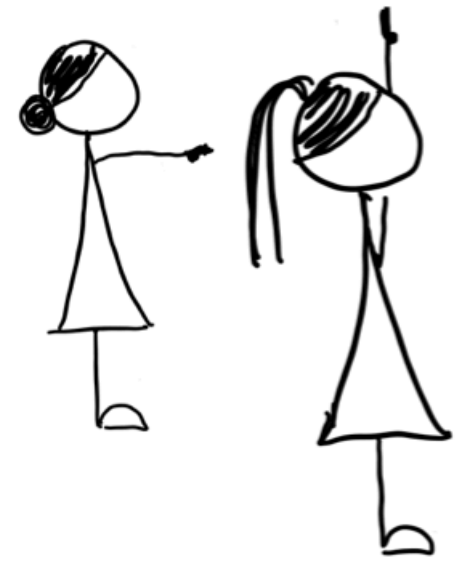


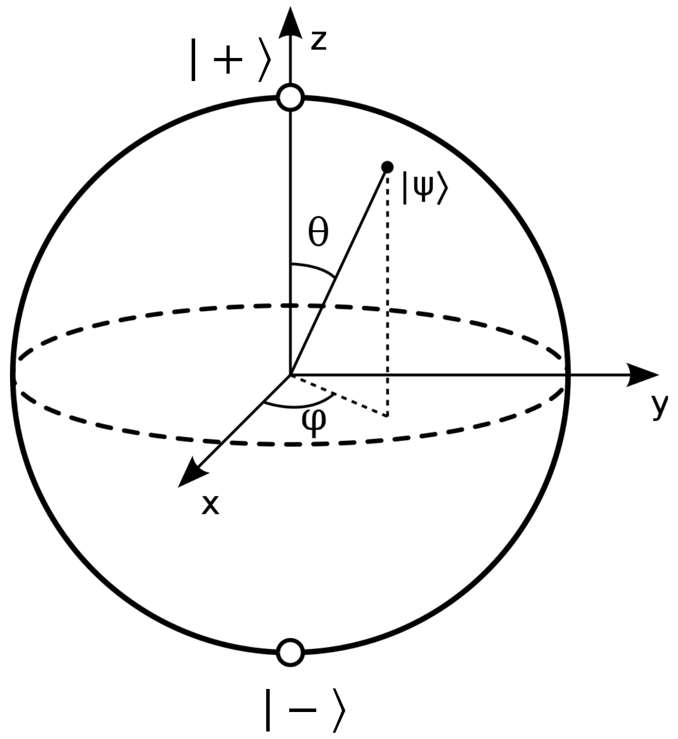


Quantum State Vector

$$\begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix}$$

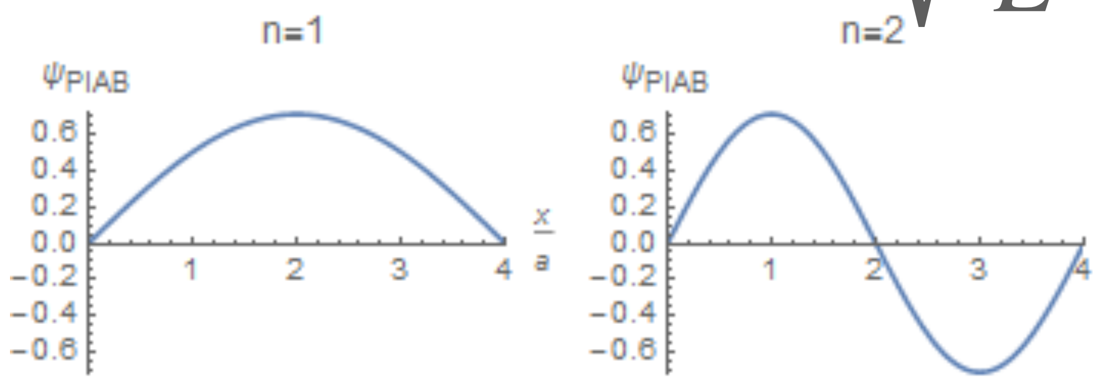
$$1/\sqrt{2}\,|+\rangle + i/\sqrt{2}\,|-\rangle$$

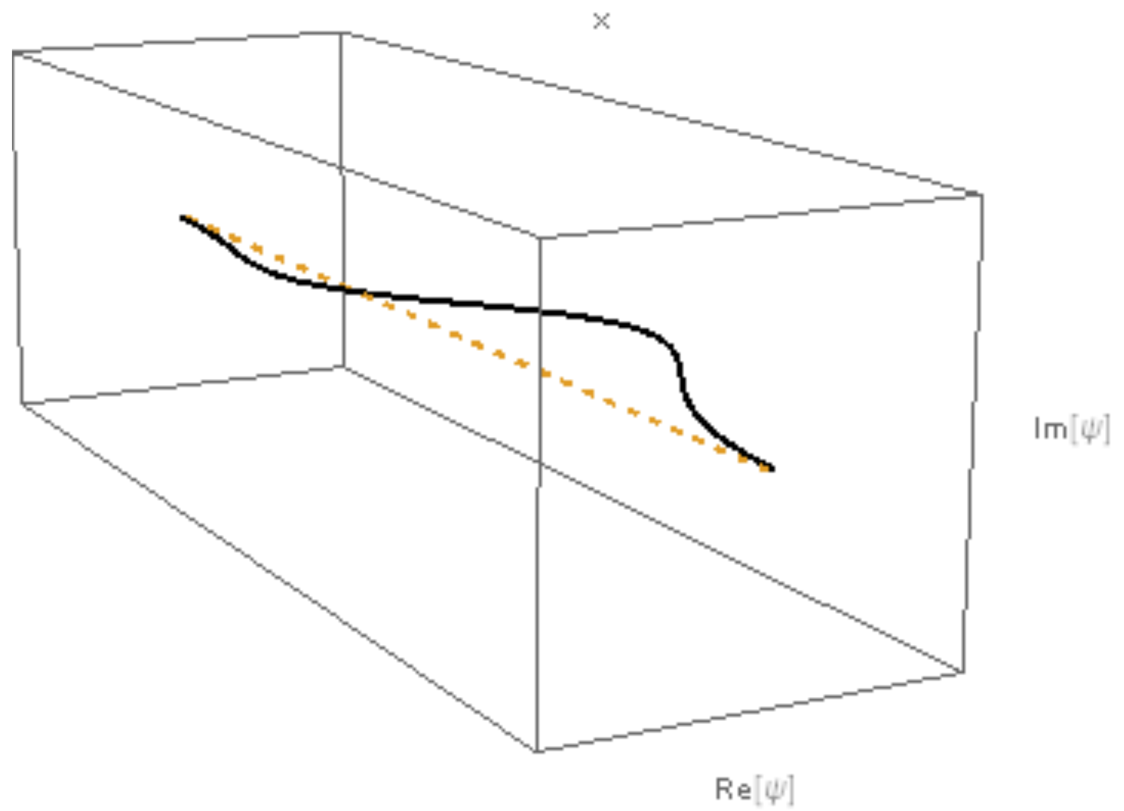




Spin Up

$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$





Elizabeth Gire
Oregon State University

Colloquium 11/7/2023

What I’m going to talk about…

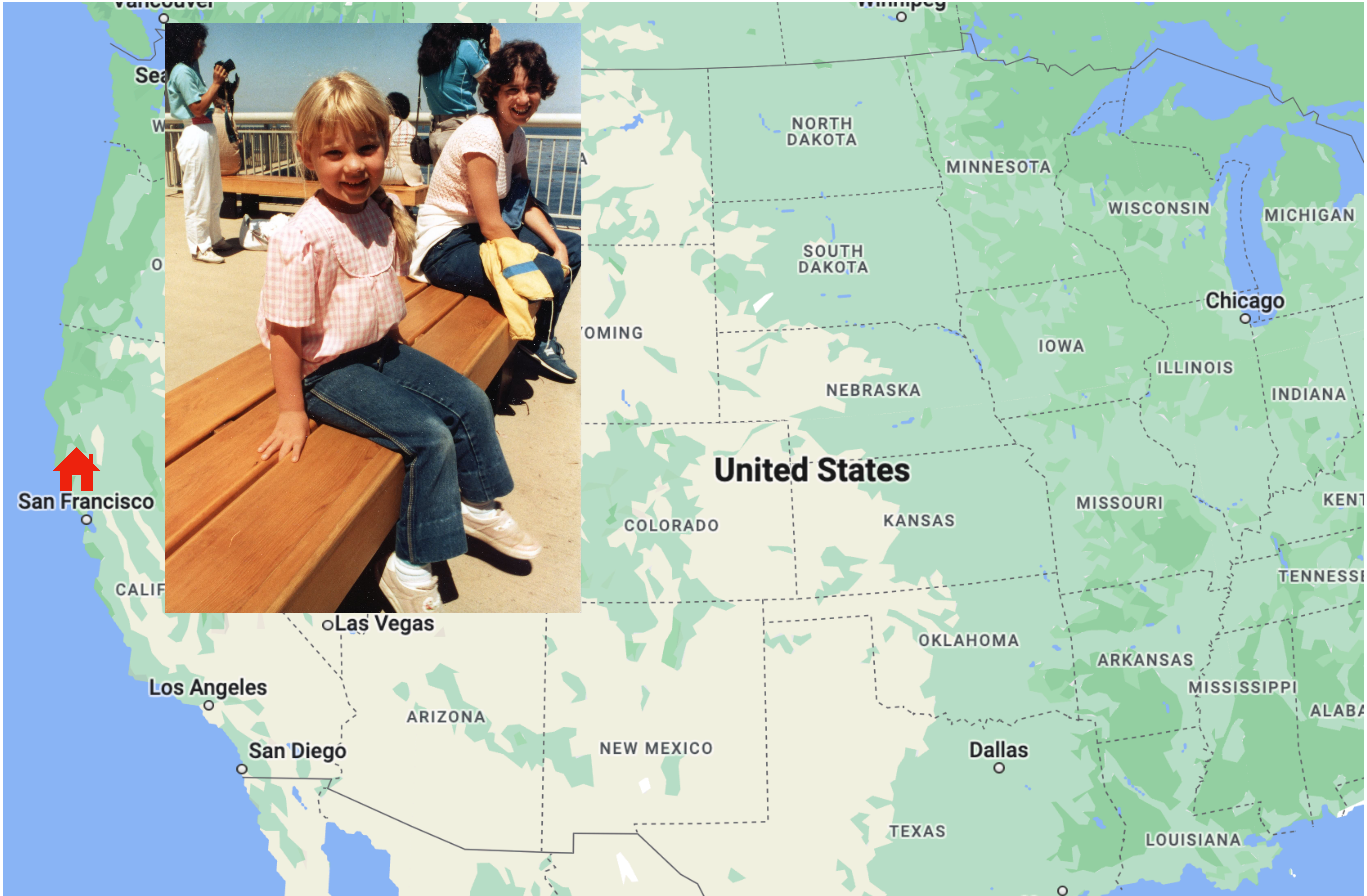
A Bit About My Story

Thinking with External Representations

Discrete & Continuous Observables

Quantum Representations for
Understanding the Connection Between
Discrete and Continuous Observables

A Bit About My Story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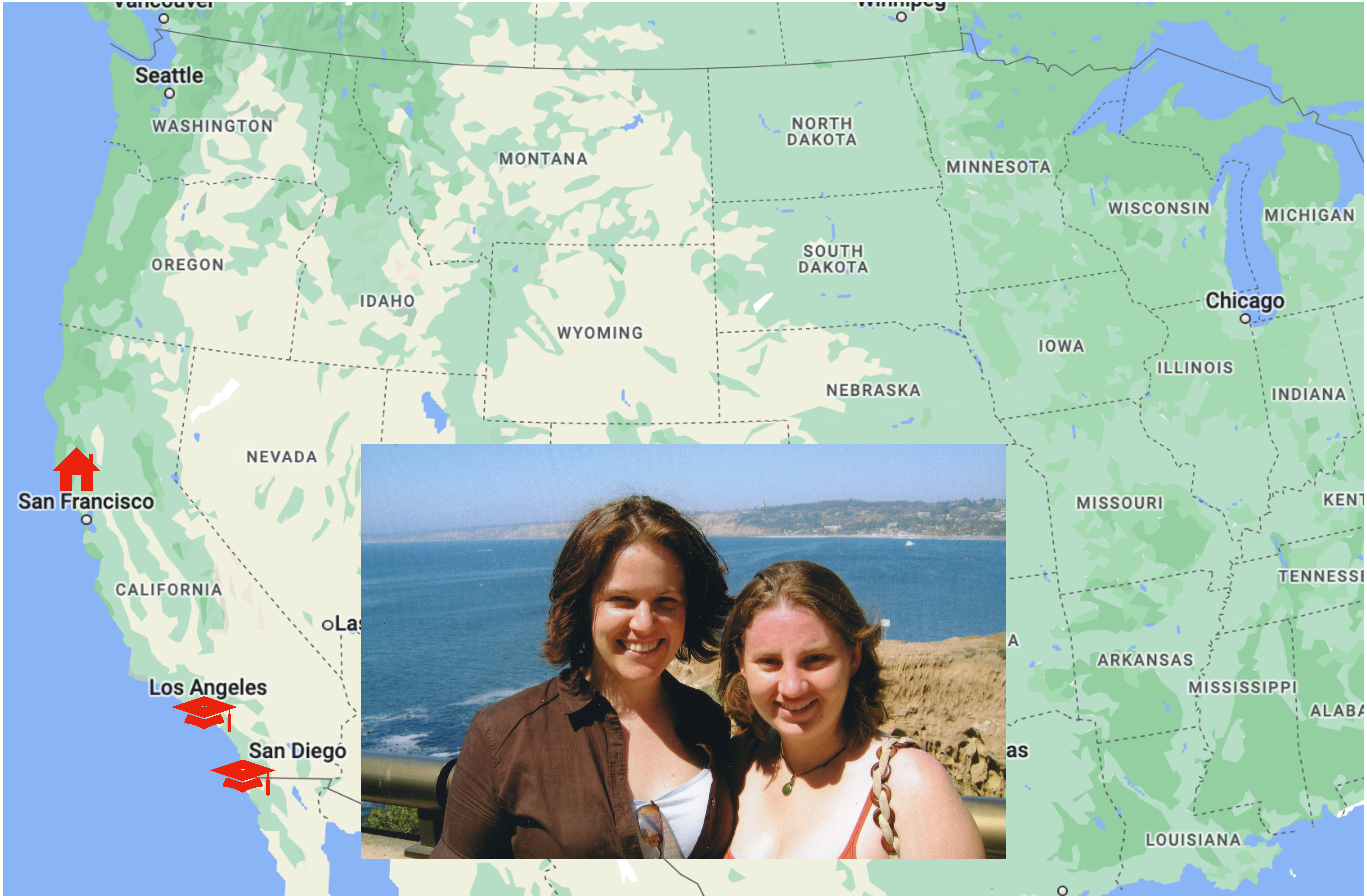
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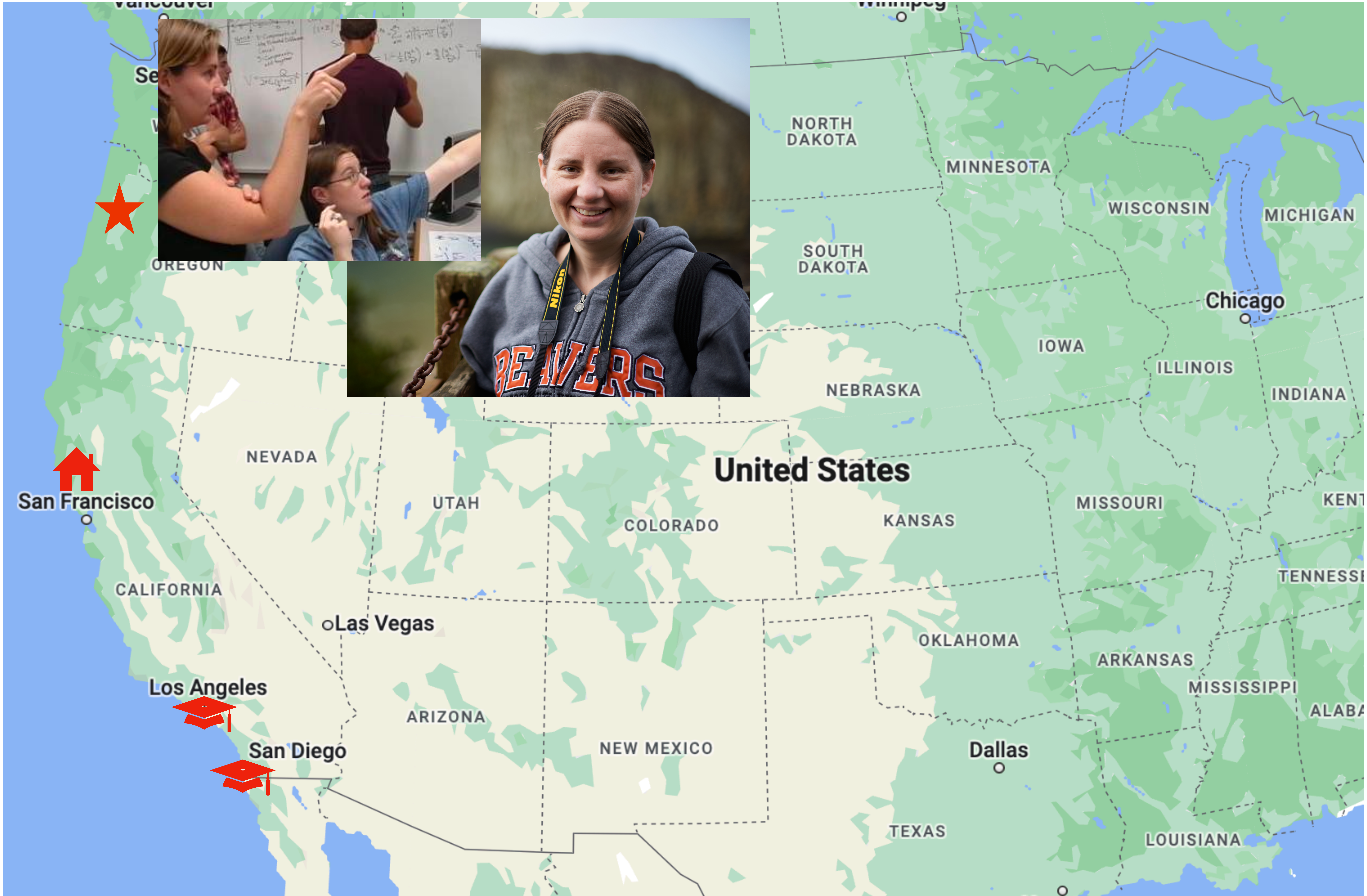
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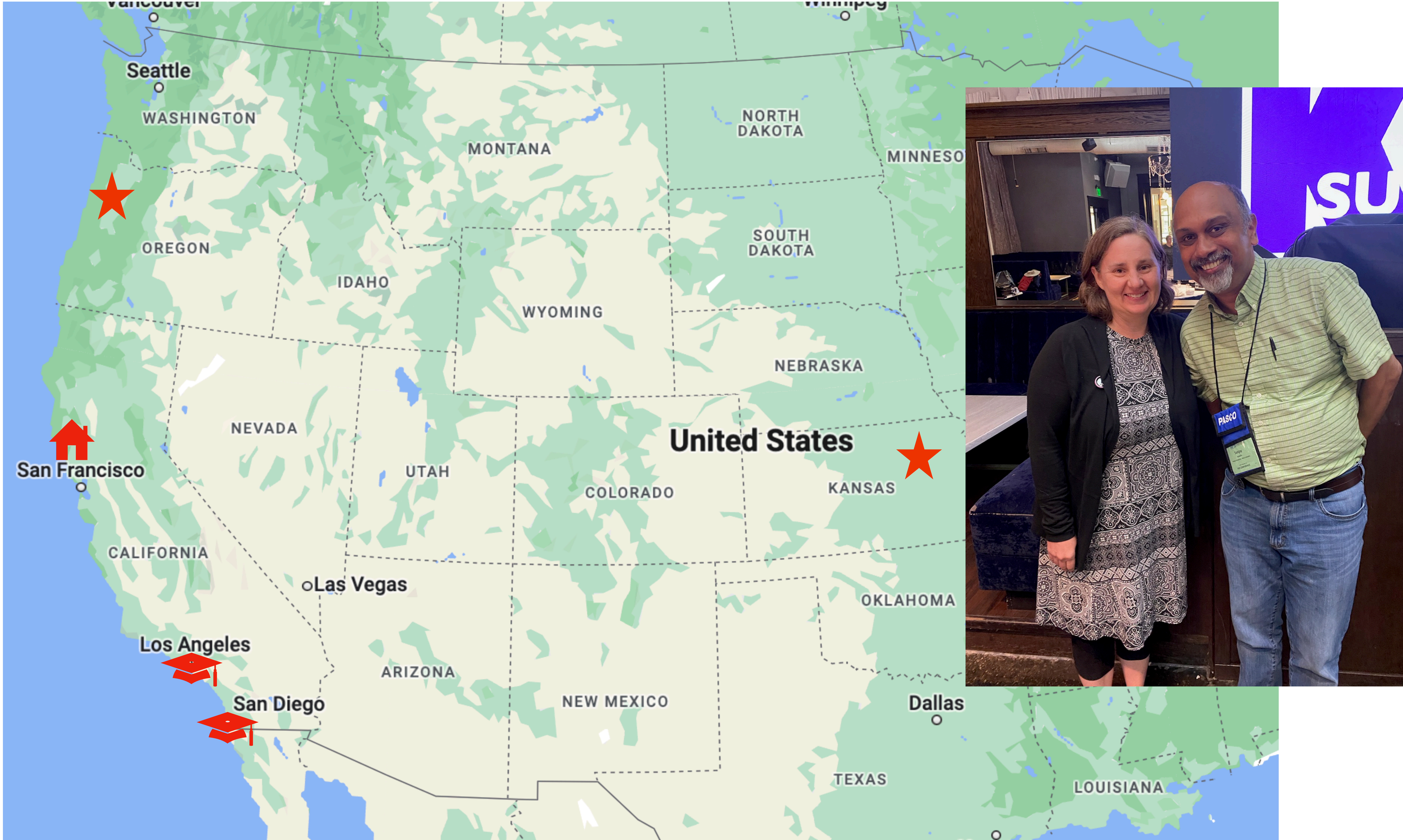
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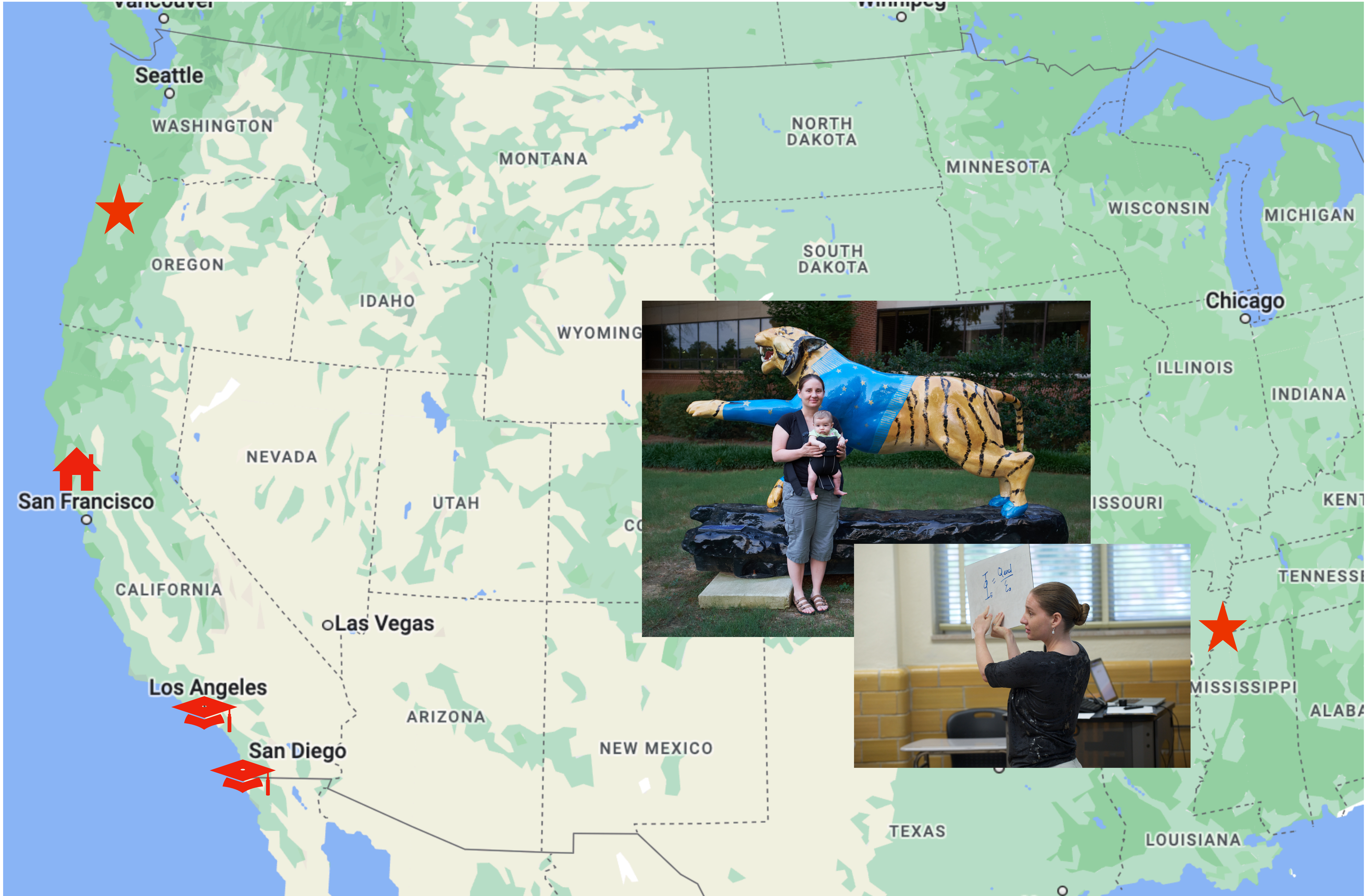

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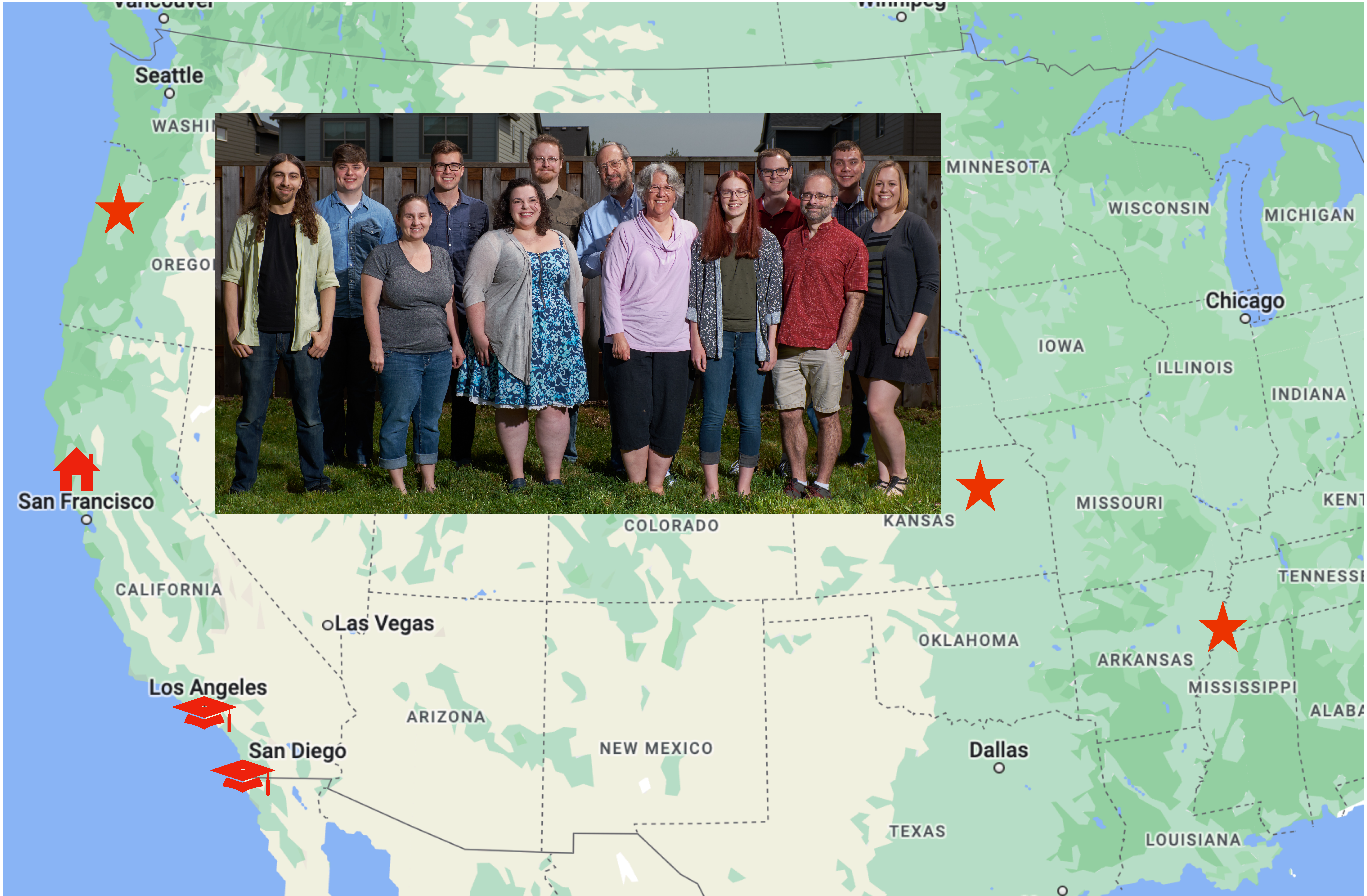
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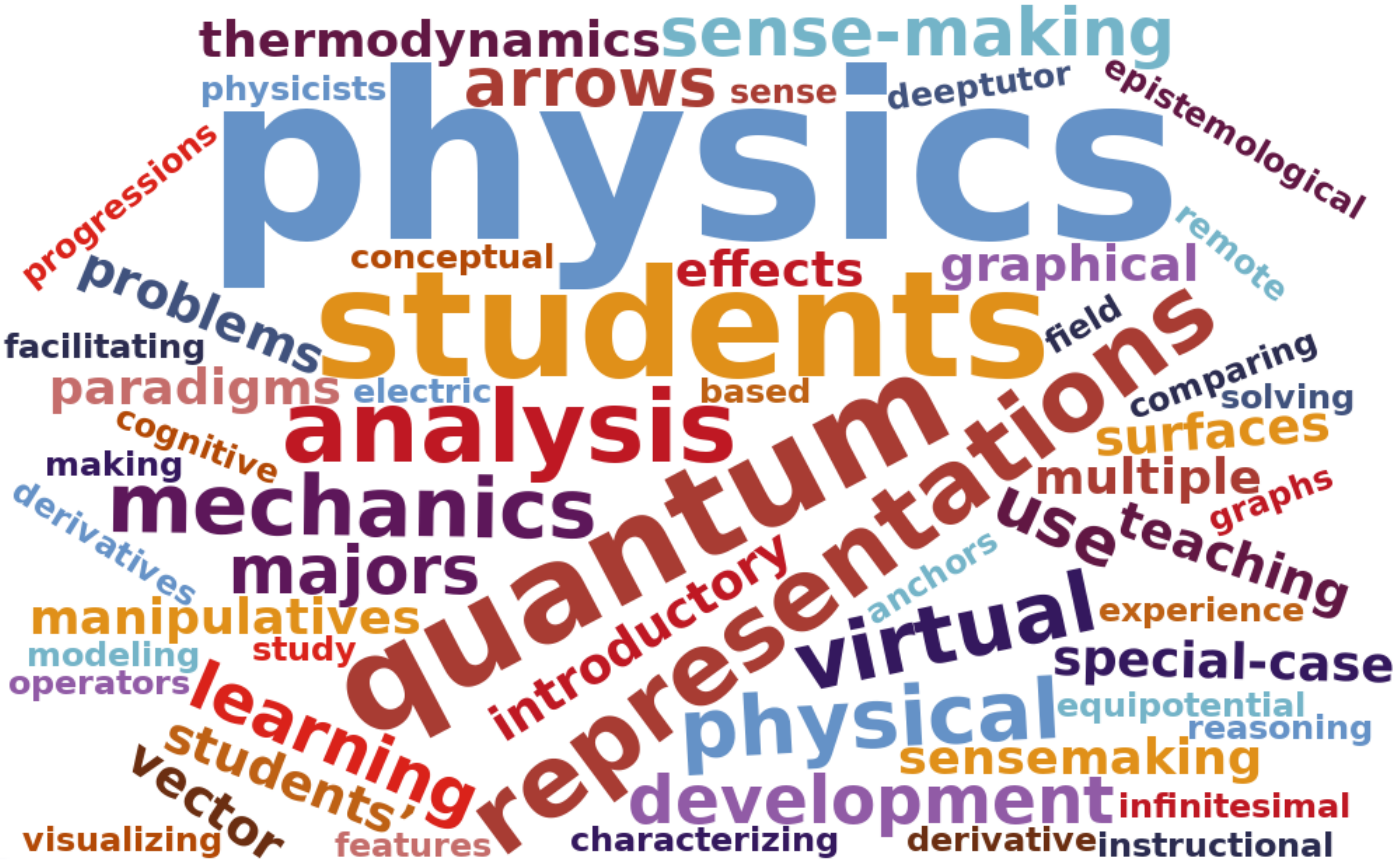
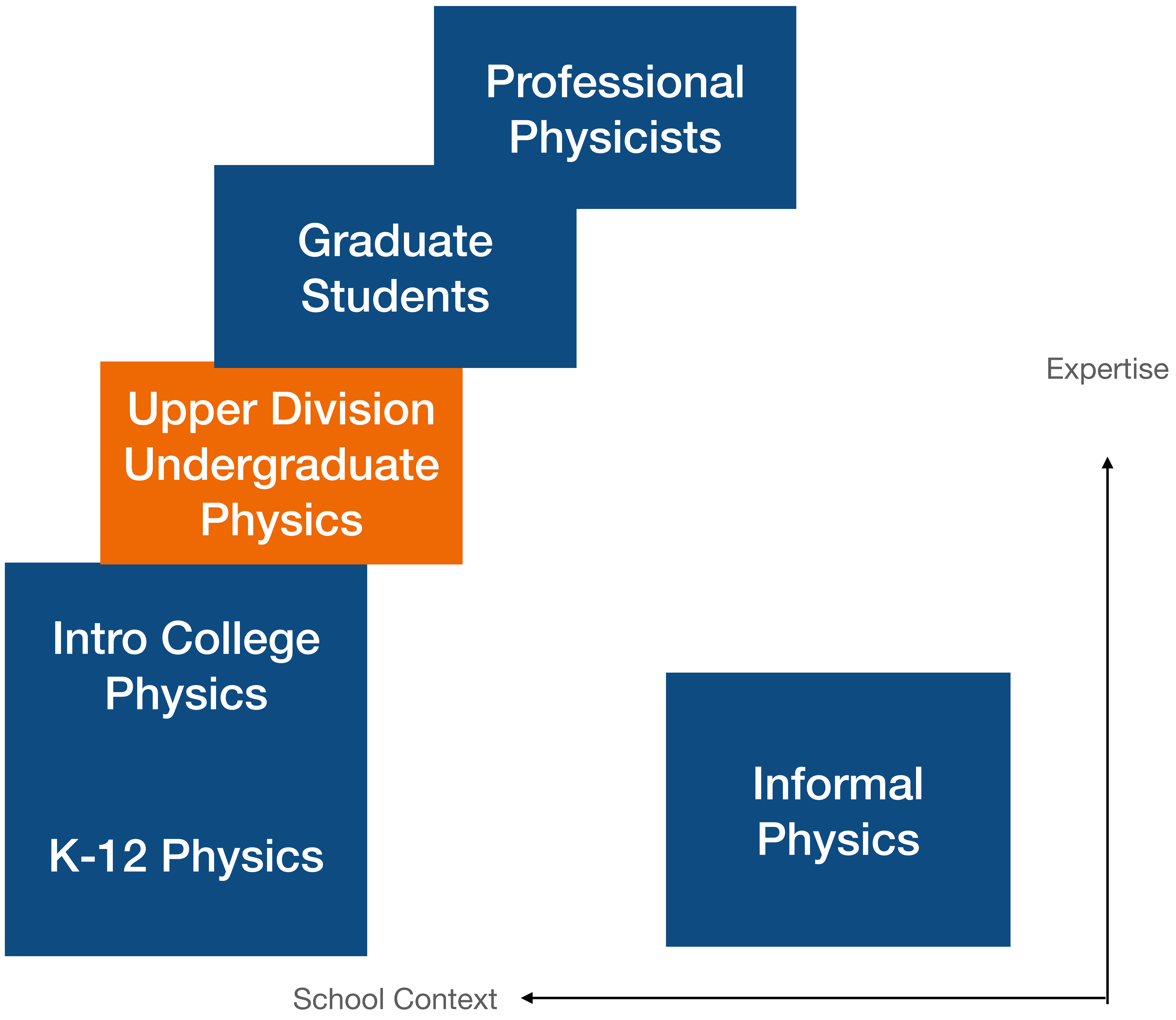


Guiding Research Aim

How can we best support the learning of the next generation of physicists?



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Emily Van Zee
Paul Emigh

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Jason Ward
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Luke Nearhood

Former Members
Christian Solorio
Jonathan Alfson
Kelby Hahn
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Michael Vignal
MacKenzie Lenz
Greg Mulder
Emily Smith
Len Cerny
Kerry Brown
Grant Sherer
Ian Founds
Mesa Walker
Mary Bridget Kustusch
Rabindra Bajracharya

NSF DUE Grant Nos. 9653250, 0231194, 0618877, 0837829, 1023120, 1141330, 1323800,1612480,1836603,1836604

Thinking with
External Representations

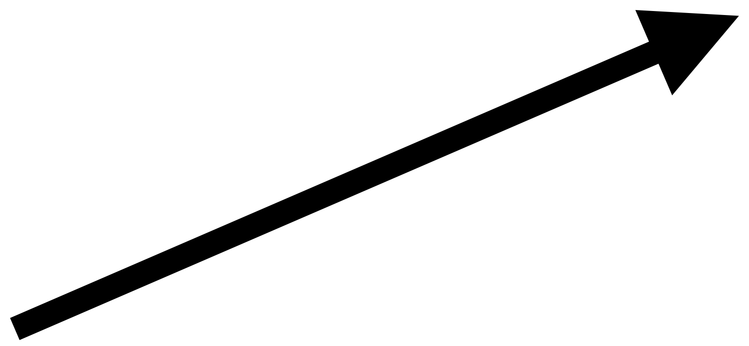
External Representations

Organized Information

Medium

Referent

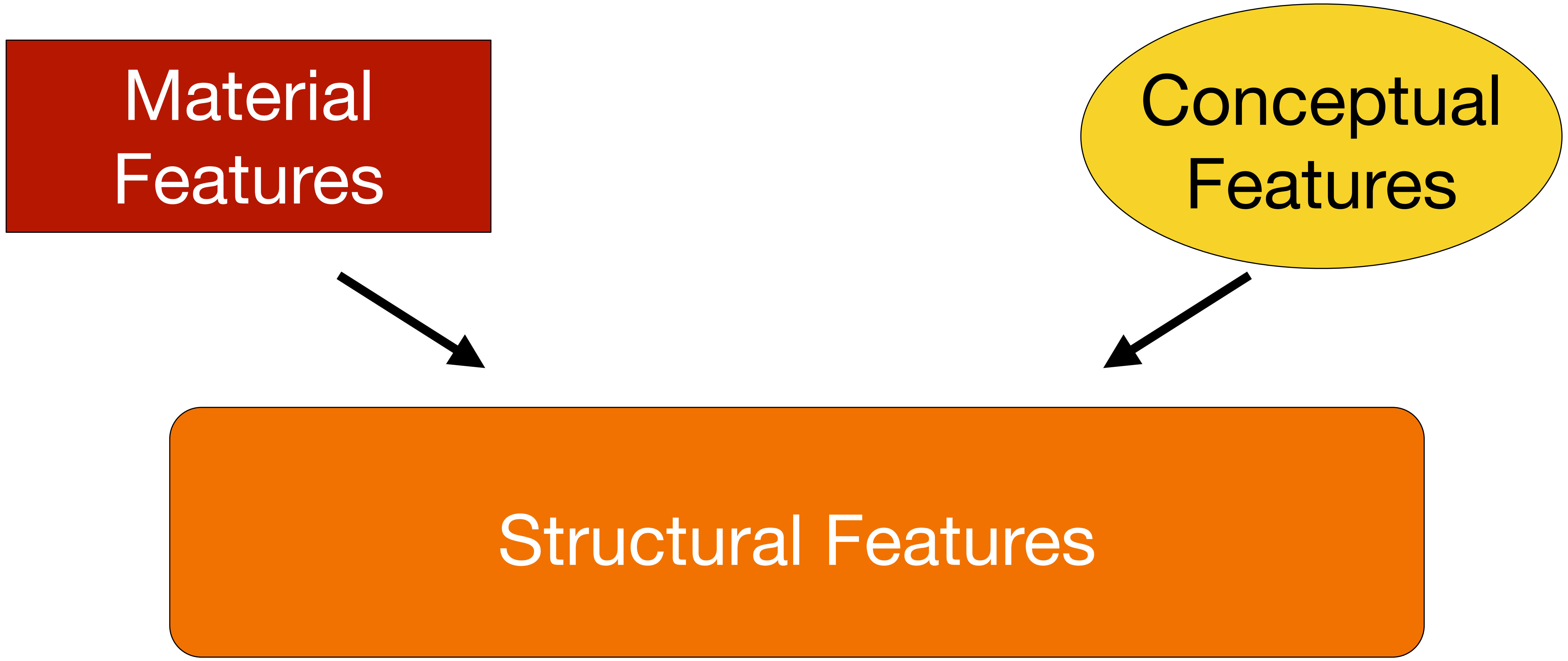
$\vec{v}$



Gire & Price, *Phys. Rev. PER*, 2015

Gire, et al., In review

External Representations



Literalness Externalization Individuation

Gire & Price, PRPER, 2015
Gire, et al., In Review

Metaphors for the Brain

Brain-as-Computer

Brain-as-Muscle

Brain-as-Beaver*

A. M. Paul, *The Extended Mind*, 2021

https://www.youtube.com/watch?v=-ImdlZtOU80

Brain as Beaver

Thought draws on **resources external to the brain**

The resources **affect the nature and quality** of thought

Thinking well is a state that depends on those resources and the knowledge of how to use them well.

A. M. Paul, *The Extended Mind*, 2021

https://www.youtube.com/watch?v=-ImdlZtOU80

Brain-as-Beaver

Opportunities

"use bodily movements and gestures to understand highly conceptual subjects like science"

Embodied Cognition

"how abstract ideas can be turned into physical objects that can be manipulated and transformed in order to achieve insights and solve problems"

Situated Cognition

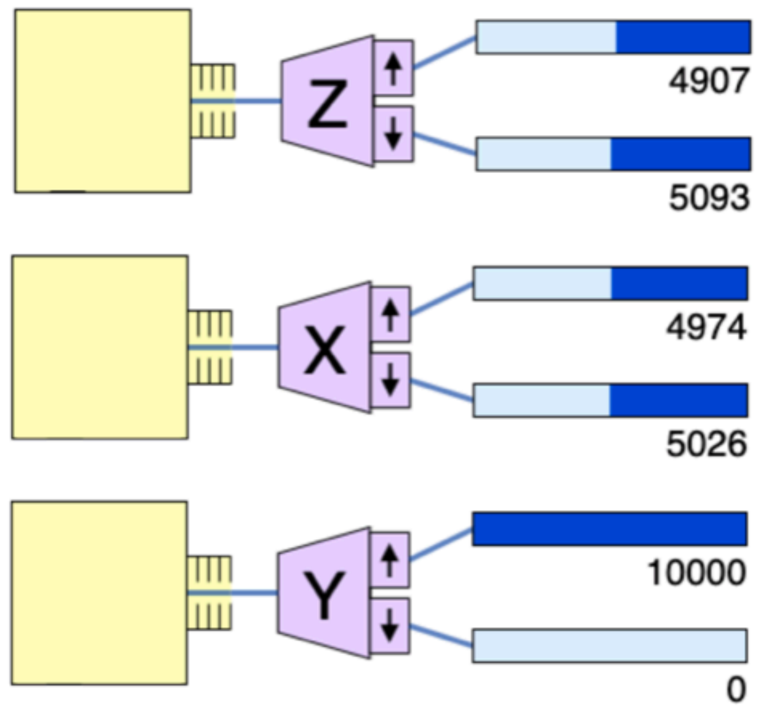
"Classroom groups and workplace teams …coached in scientifically validated methods of increasing the collective intelligence of their members"

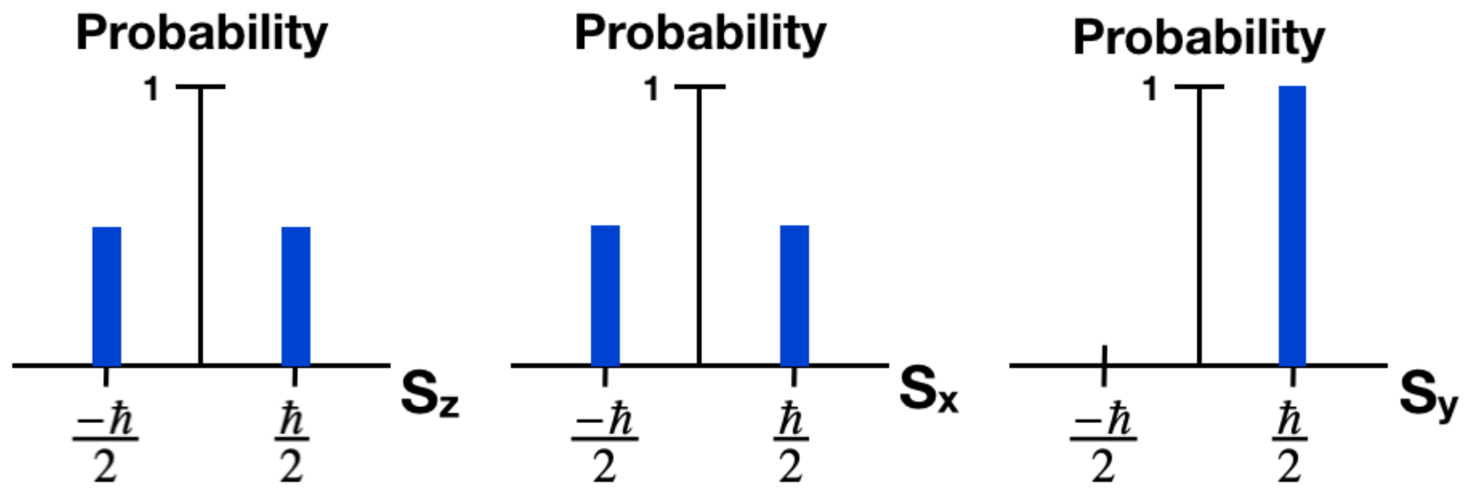
Distributed Cognition

A. M. Paul, *The Extended Mind*, 2021

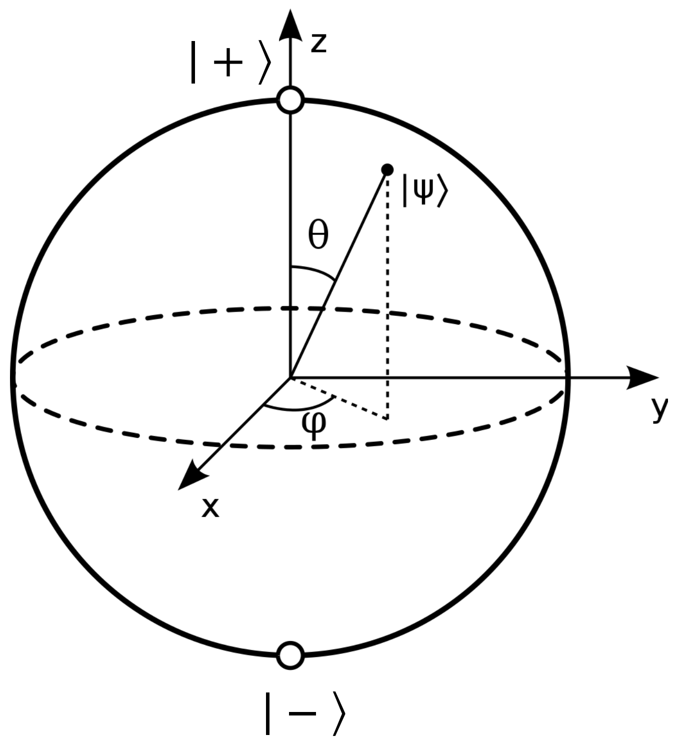
External Representations

Languages = Tools for Thinking & Communication



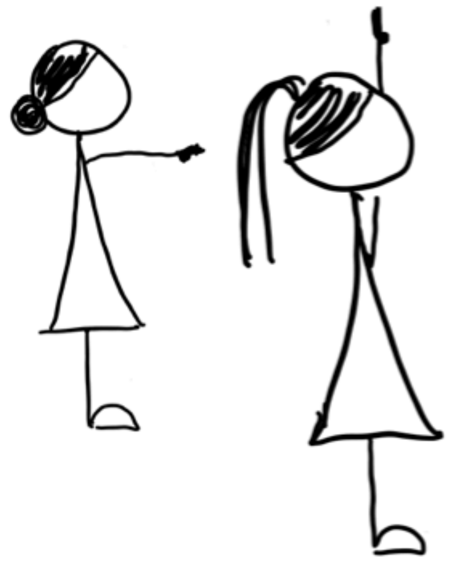


Quantum State Vector



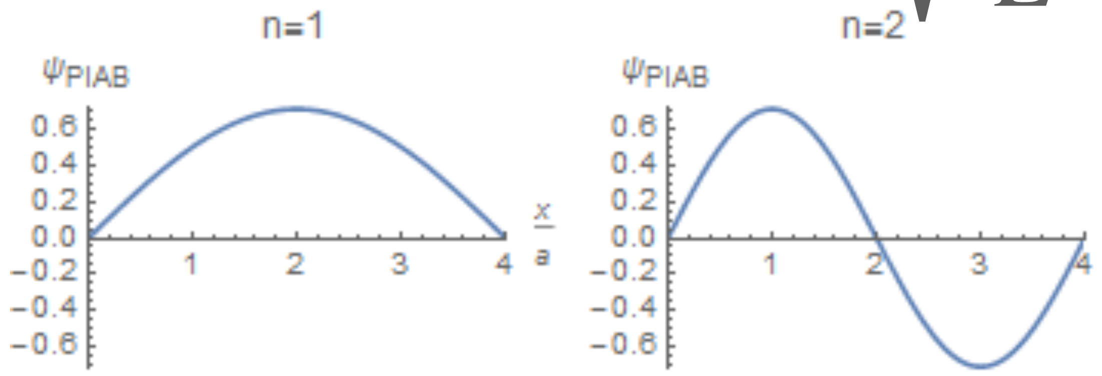
$$\begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix}$$

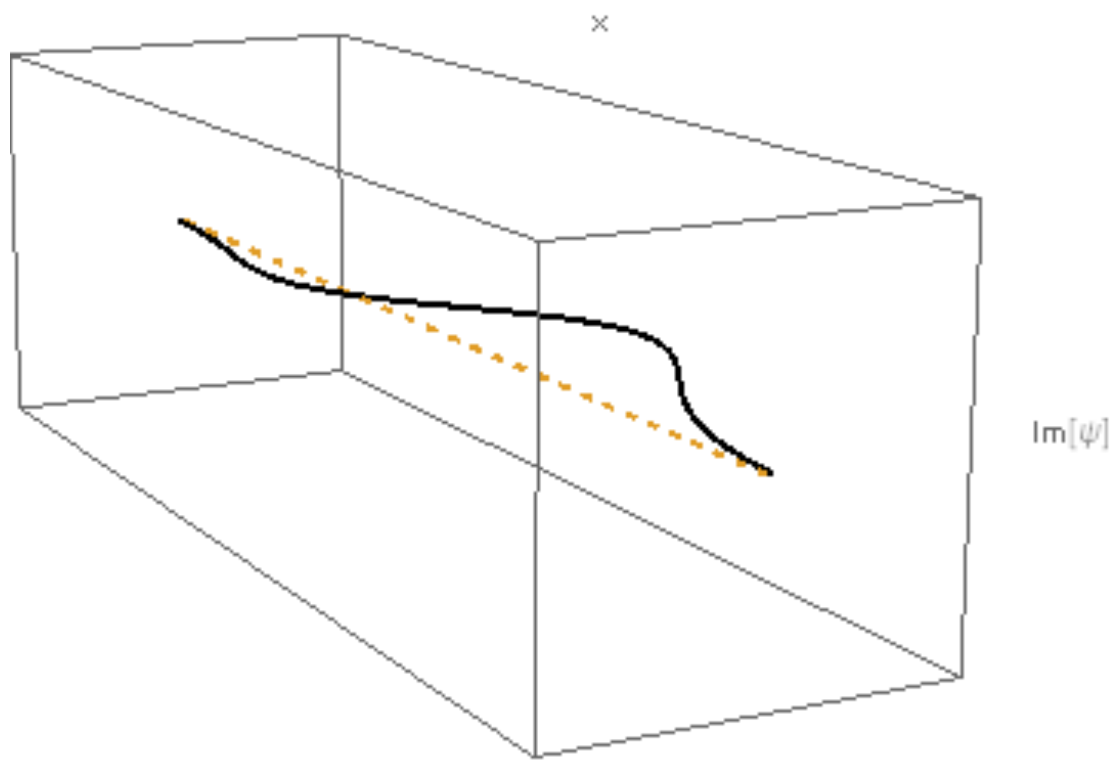
$$1/\sqrt{2}\,|+\rangle + i/\sqrt{2}\,|-\rangle$$



Spin Up

$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$





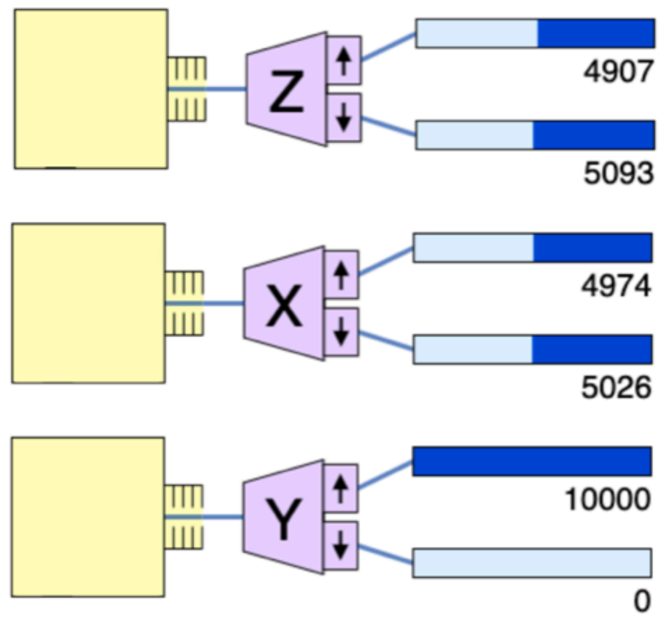
"'Concepts' as they are used in scientific communication, and in scientific work generally, are not defined by the common denominator of their representations, but by the sum, **the union of meanings implied by all these representations**…What we call abstract concepts is only a shorthand for a multimodal semiotic construction, a simultaneous and multiply articulated cluster of independent practices."

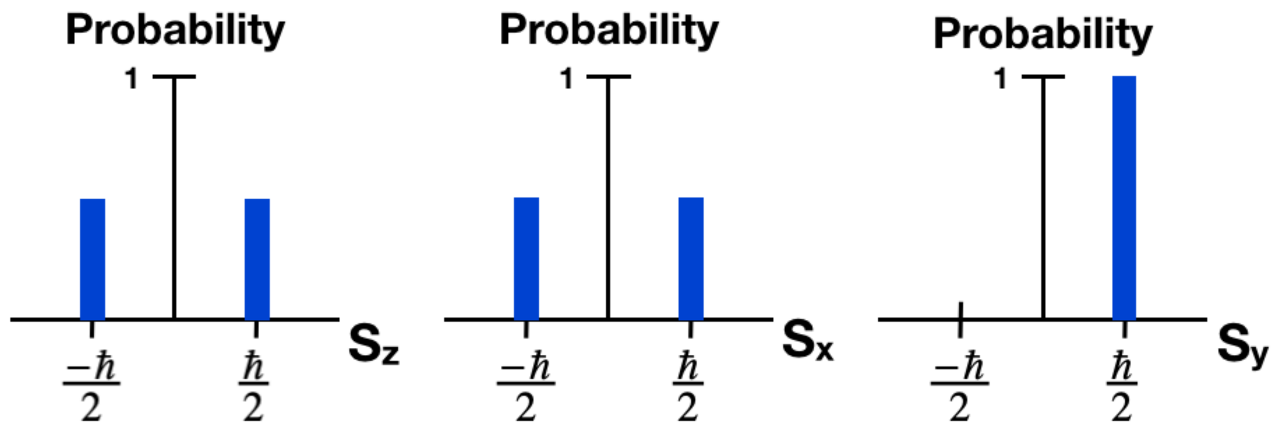
Jay Lemke
Multiplying meaning: Visual and verbal semiotics in scientific text, 1998

Instructional Context

Paradigms in Physics
Quantum Fundamentals & Central Forces Courses

- “Spins First” Approach (McIntyre textbook)
- Stern-Gerlach Simulation to explore postulates of quantum mechanics
- Emphasize Multiple Representations
- Computational lab





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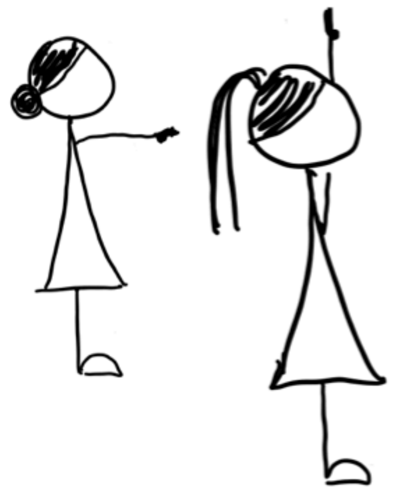
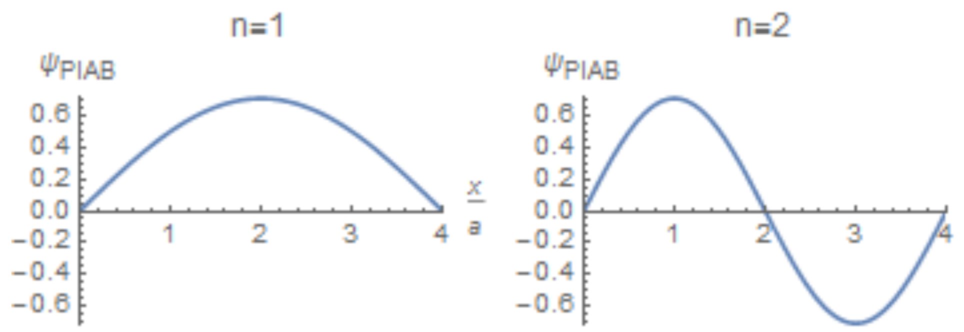
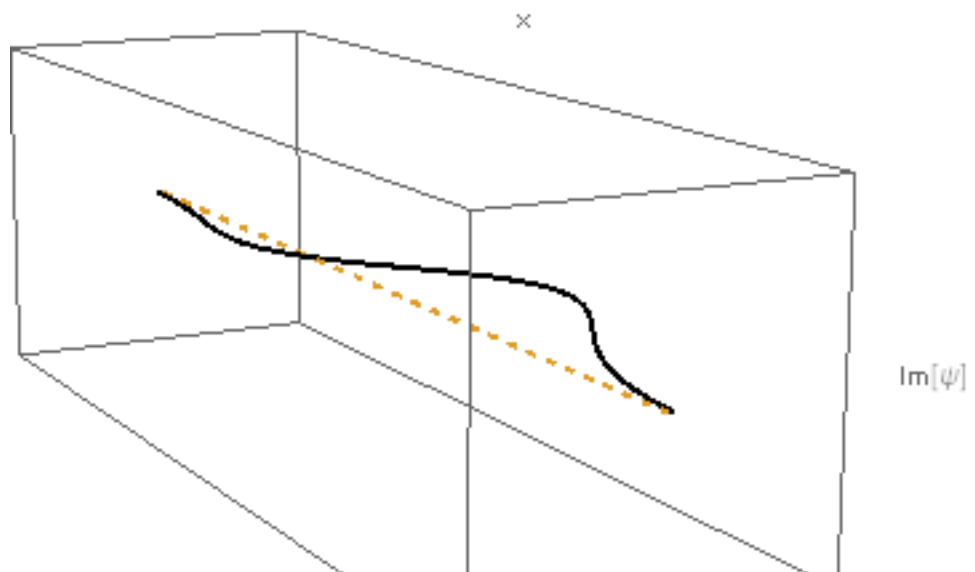
$$1/\sqrt{2}\,|+\rangle + i/\sqrt{2}\,|-\rangle$$



```
sum = 0
for x in np.arange(0, L, dx):
    sum += np.conj(Phi(n, x))*Psi(x)*dx
```

$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$



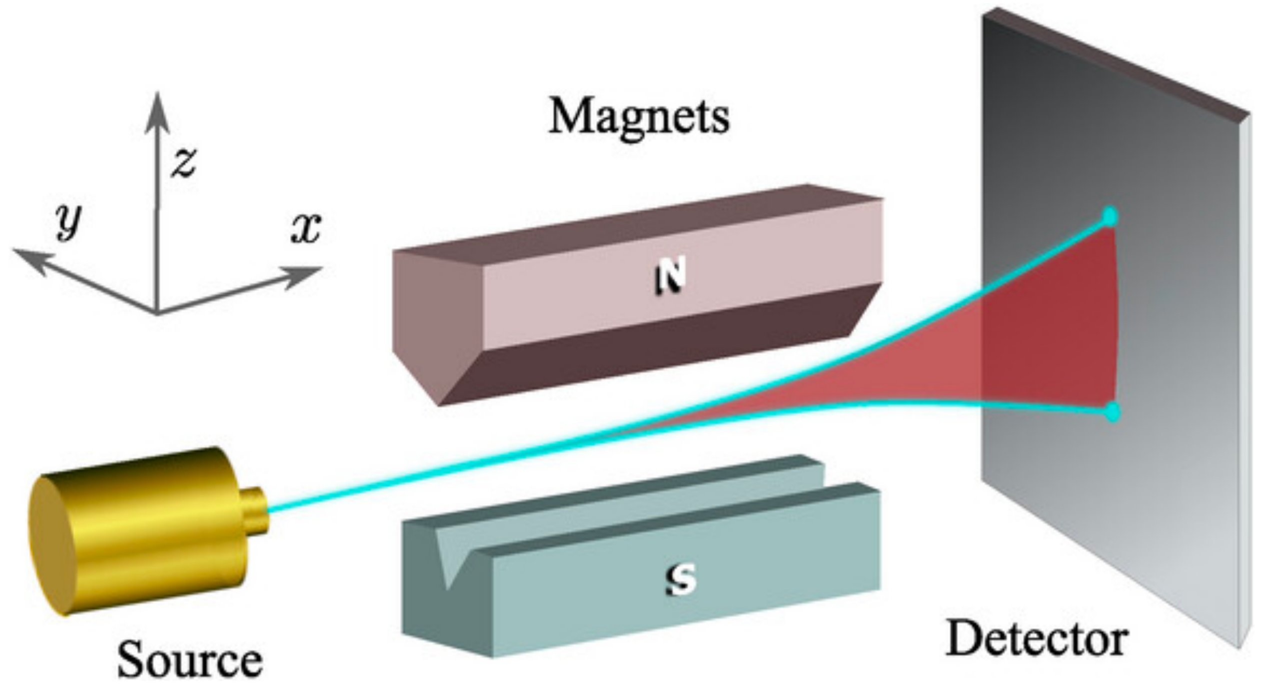


Emigh, et al., *Phys. Rev. PER*, 2020
Manogue, et al., *Am. J. Phys.*, 2001

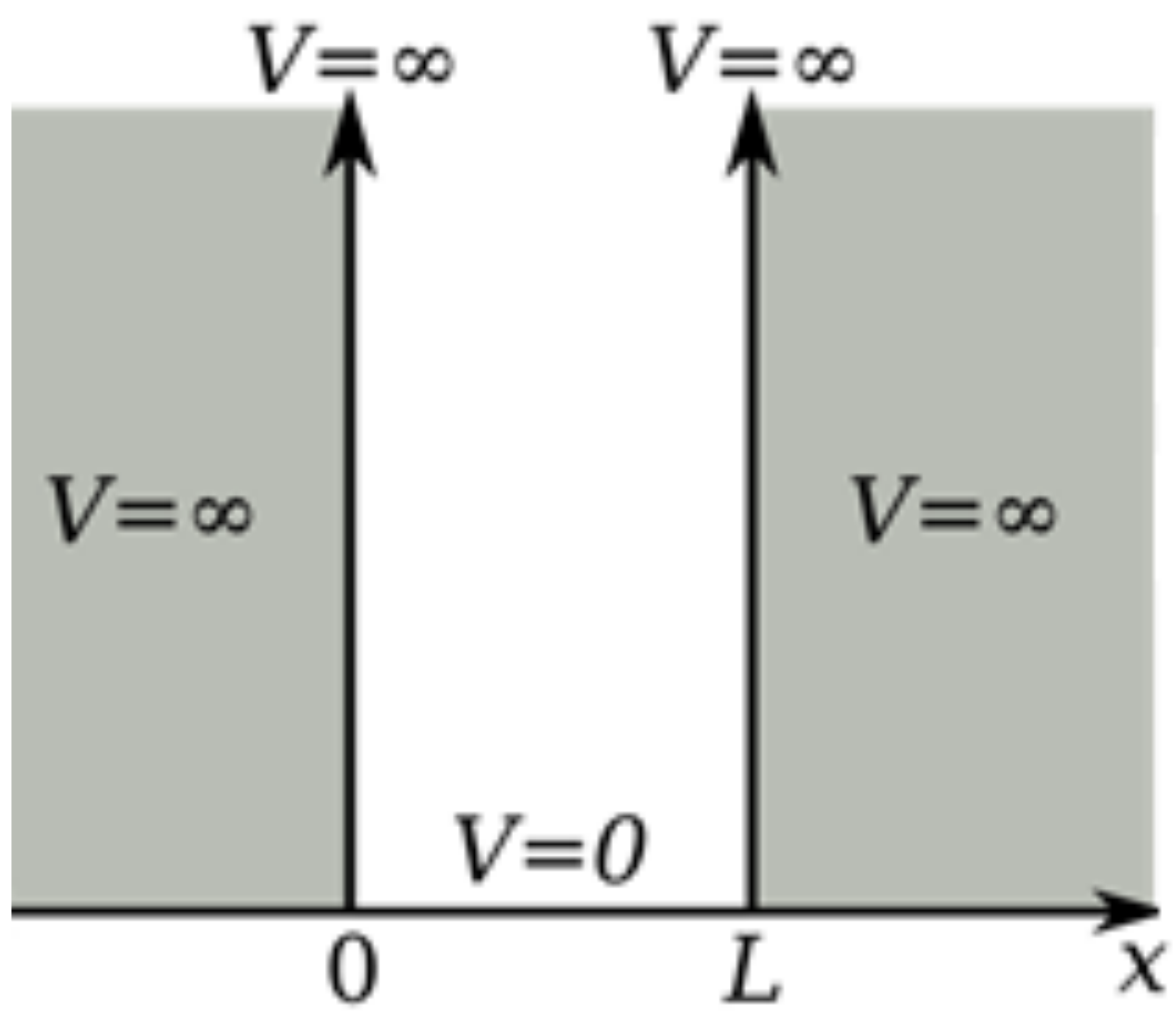
Discrete

Continuous

Motivation



Spin



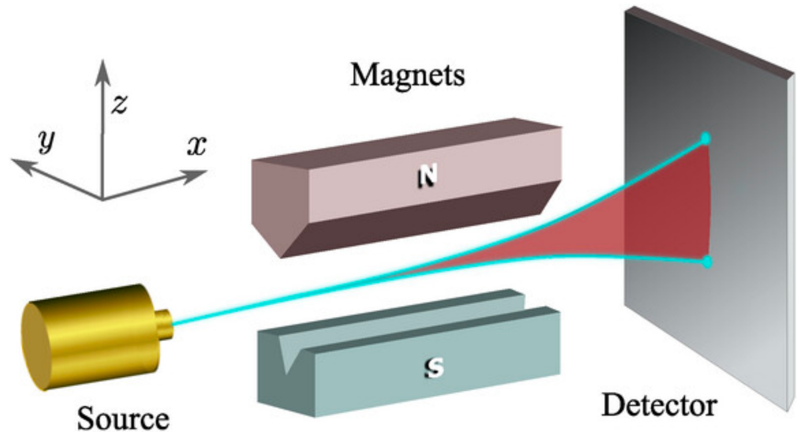
Particle in a Box

Discrete & Continuous Observables

Discrete

Continuous

Spin Component

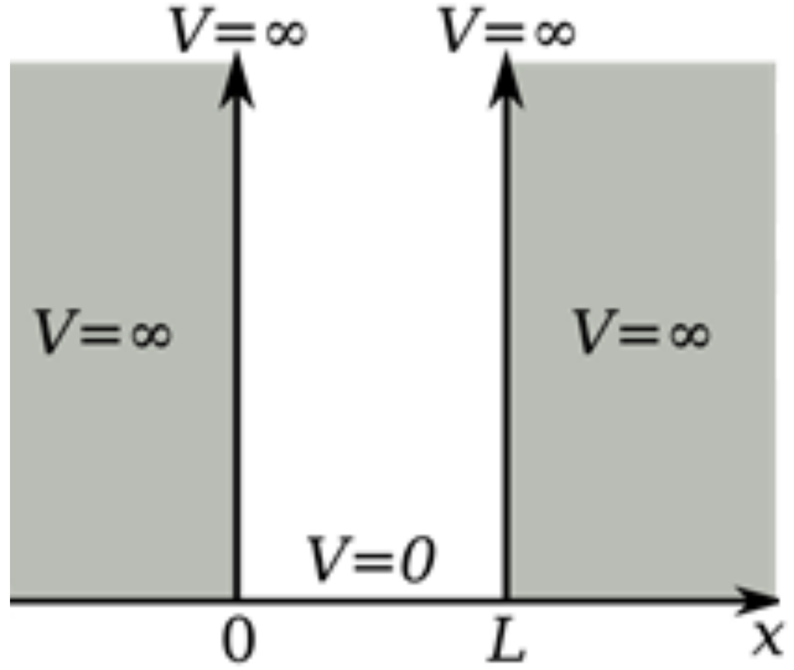


$$S_z = \pm\frac{\hbar}{2}$$

$$|+\rangle_z \quad |-\rangle_z$$

$$\mathscr{P}\left(S_z = +\frac{\hbar}{2}\right) = \left|{}_z\langle + | \psi_s\rangle\right|^2$$

Energy of Particle in a Box



$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$|n\rangle$$

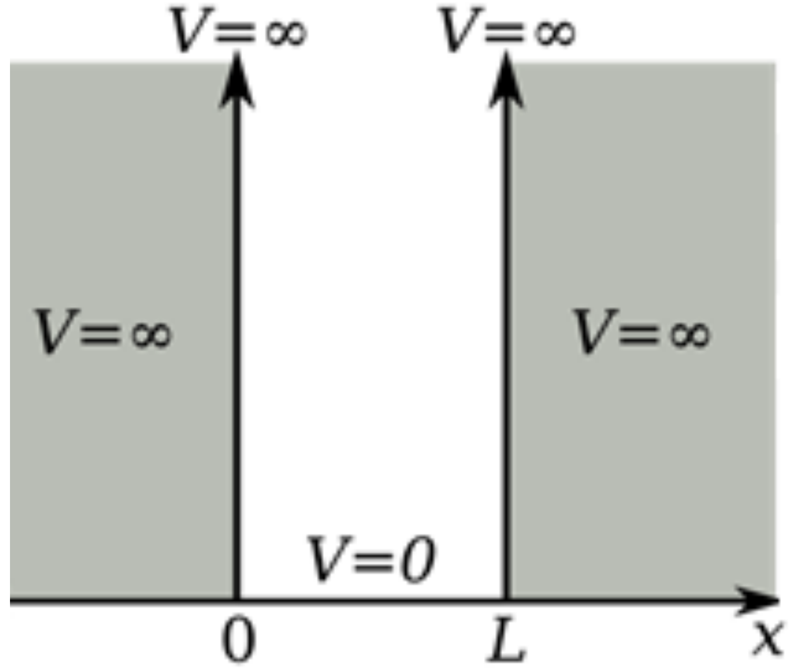
$$|n\rangle \doteq \phi_n(x)$$

$$\mathscr{P}\left(E = E_n\right) = \left|\langle n | \psi_p\rangle\right|^2$$

$$\mathscr{P}\left(E = E_n\right) = \left|\int_0^L \phi_n^*(x)\,\psi_p(x)\,dx\right|^2$$

$$\mathscr{P}\left(E_1 < E < E_{10}\right) = \sum_{n=1}^{10}\left|\langle n | \psi_p\rangle\right|^2$$

Position of Particle in a Box



$$0 \le x \le L$$

$$|x\rangle$$

$$|\psi_p\rangle \doteq \psi_p(x)$$

$$\mathscr{P}\left(a < x < b\right) = \int_a^b |\psi_p(x)|^2\,dx$$

Going from Kets to Wavefunctions:

Our Former Approach

“rules for translating bra-ket formulae to wave function formulae”

1) Replace ket with wave function	$\lvert\psi\rangle \rightarrow \psi(x)$
2) Replace bra with wave function conjugate	$\langle\psi\rvert \rightarrow \psi^*(x)$
3) Replace braket with integral over all space	$\langle\,\vert\,\rangle \rightarrow \int_{-\infty}^{\infty} dx$
4) Replace operator with position representation	$\hat{A} \rightarrow A(x)$

McIntyre “Quantum Mechanics”, p. 116

Calculating Probabilities

$$\mathscr{P}(E_n) = \left|\langle n|\psi\rangle\right|^2 \quad \Rightarrow \quad \mathscr{P}(E_n) = \left|\int \phi_n^*(x)\,\psi(x)\,dx\right|^2$$

$$\mathscr{P}(E_n) \neq \int \left|\phi_n^*(x)\,\psi(x)\right|^2 dx$$

$$\mathscr{P}(E_n) \neq \int \phi_n^*(x)\,\psi(x)\,dx$$

$$\mathscr{P}(a < x < b) = \int_a^b \left|\psi(x)\right|^2 dx$$

$$= \int_a^b \psi^*(x)\,\psi(x) dx$$

Riihiluoma, et al, PERC, 2022
Singh & Marshman, AJP, 2015

Calculating Probabilities

$$\mathscr{P}(E_n) = \left| \langle n | \psi \rangle \right|^2 \quad \Rightarrow \quad \mathscr{P}(E_n) = \left| \int \phi_n^*(x)\, \psi(x)\, dx \right|^2$$

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Riihiluoma, et al, PERC, 2022
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Calculating Probabilities

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$$= \int_a^b \psi^*(x)\,\psi(x)dx$$

$$\mathscr{P}(a < x < b) \neq \left|\int \psi_n^*(x)\,\psi(x)\,dx\right|^2$$

$$\mathscr{P}(x_0) \neq \left|\langle x_0|\psi\rangle\right|^2 = \left|\psi(x_0)\right|^2$$

Riihiluoma, et al, *PERC*, 2022
Singh & Marshman, *AJP*, 2015

Quantum Representations

for Understanding the Connection Between Discrete and Continuous Observables

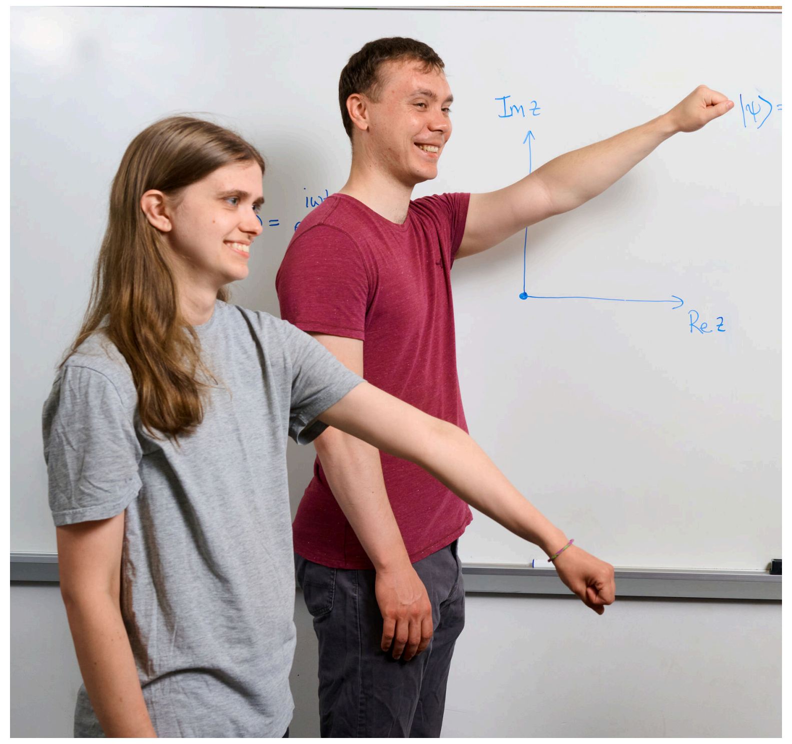
- ➡ Python Code

```
sum = 0
for x in np.arange(0, L, dx):
    sum += np.conj(Phi(n, x))*Psi(x)*dx
```

- ➡ Special forms of 1

$$\sum_n |n\rangle\langle n| = 1 \qquad \int_x |x\rangle\langle x|\, dx = 1$$

- ➡ Arms Representation of Complex Numbers



Christian Solorio

Grant Sherer

Kelby Hahn

Adam Frye

Python Code

Inner Products

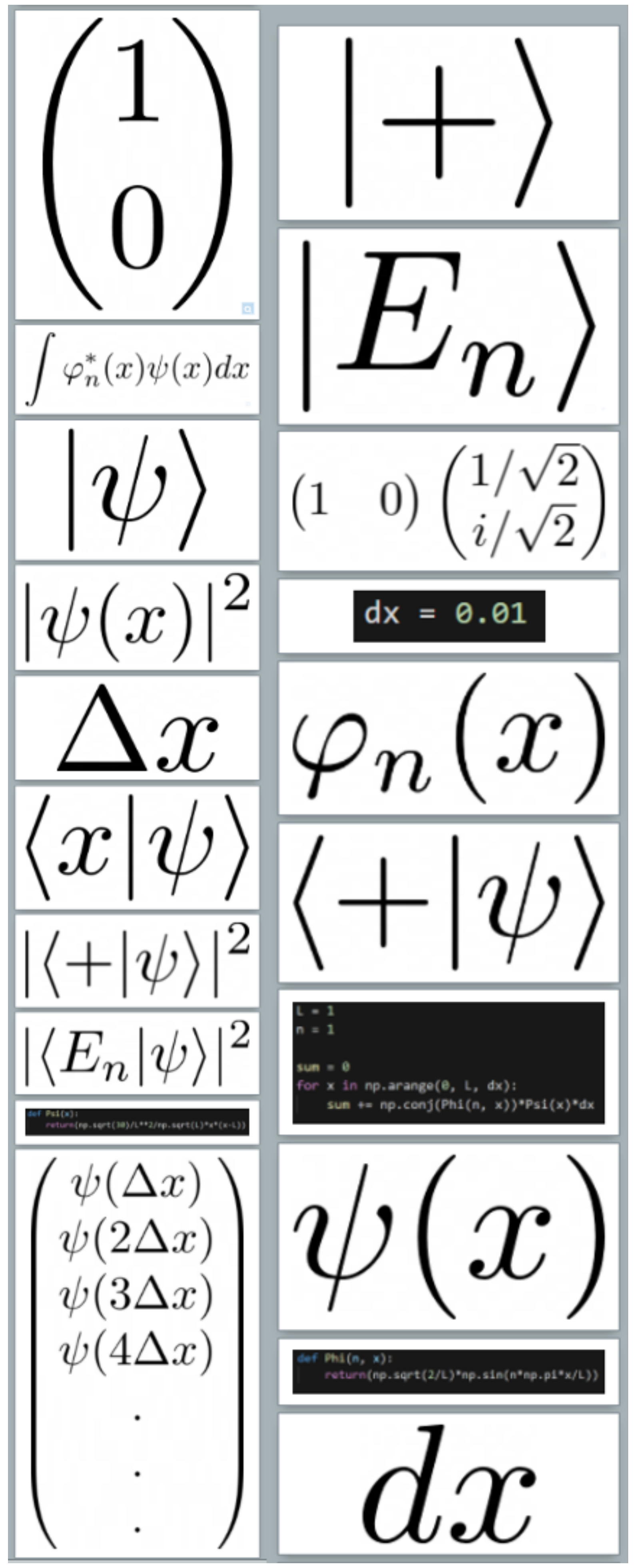
$$\langle \phi_n | \psi \rangle$$

```
sum = 0
for x in np.arange(0, L, dx):
    sum += np.conj(Phi(n, x))*Psi(x)*dx
```

$$\int \phi_n^*(x)\, \psi(x)\, dx$$

Student Understanding of Discrete & Continuous

In Quantum Mechanics



Try it!

Discreteness of Representations

- Dirac, Matrix notation are discrete
- Functions of x are continuous
- Code is discrete or continuous

Understandings of Discrete

- Single values
- Particular points

Understandings of Continuous

- Continuity of Function
- Continuity of Domain

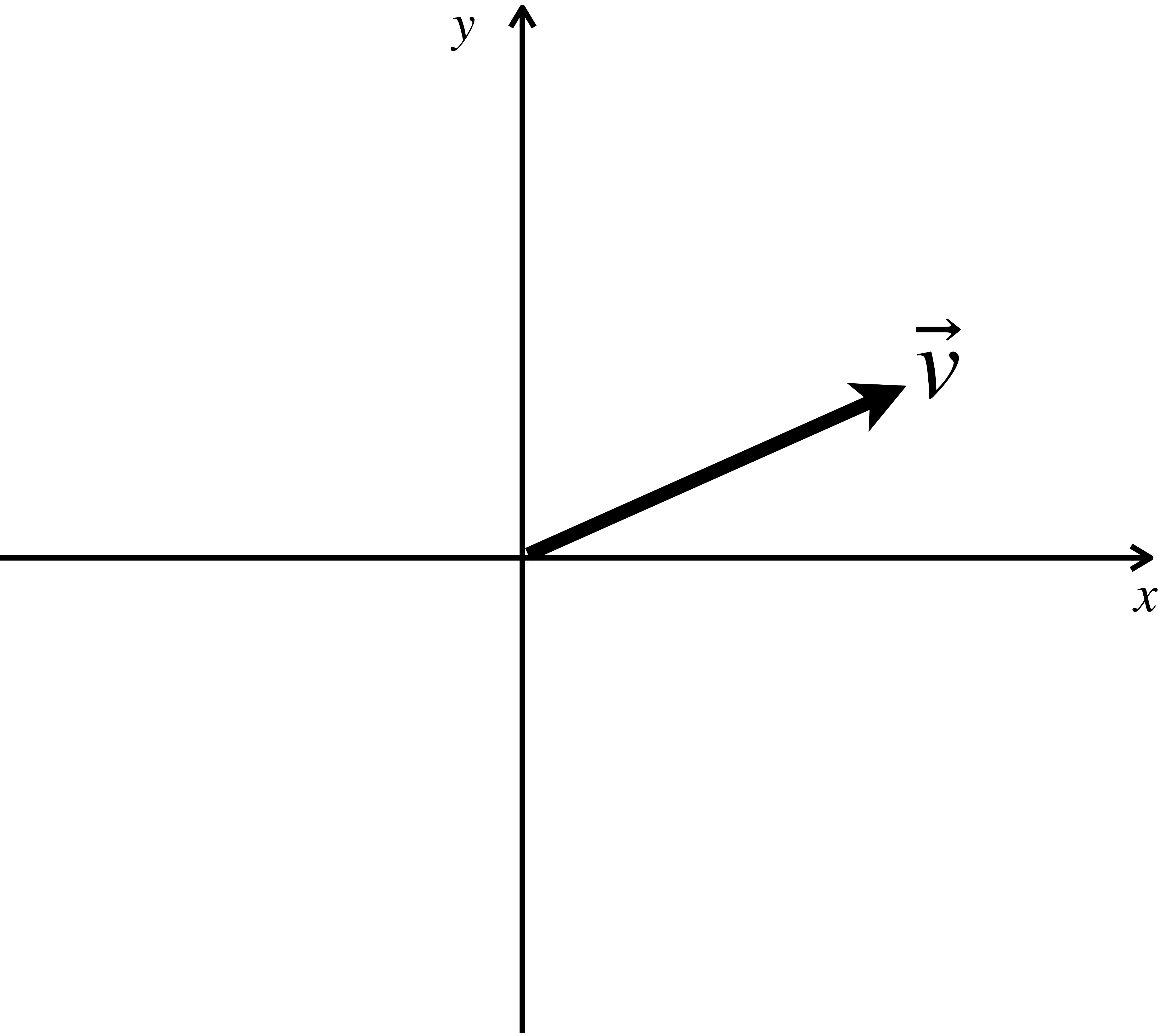
Discrete Approximations of Integrals

- Accuracy and dx size
- Limiting processes to go from summation to integration

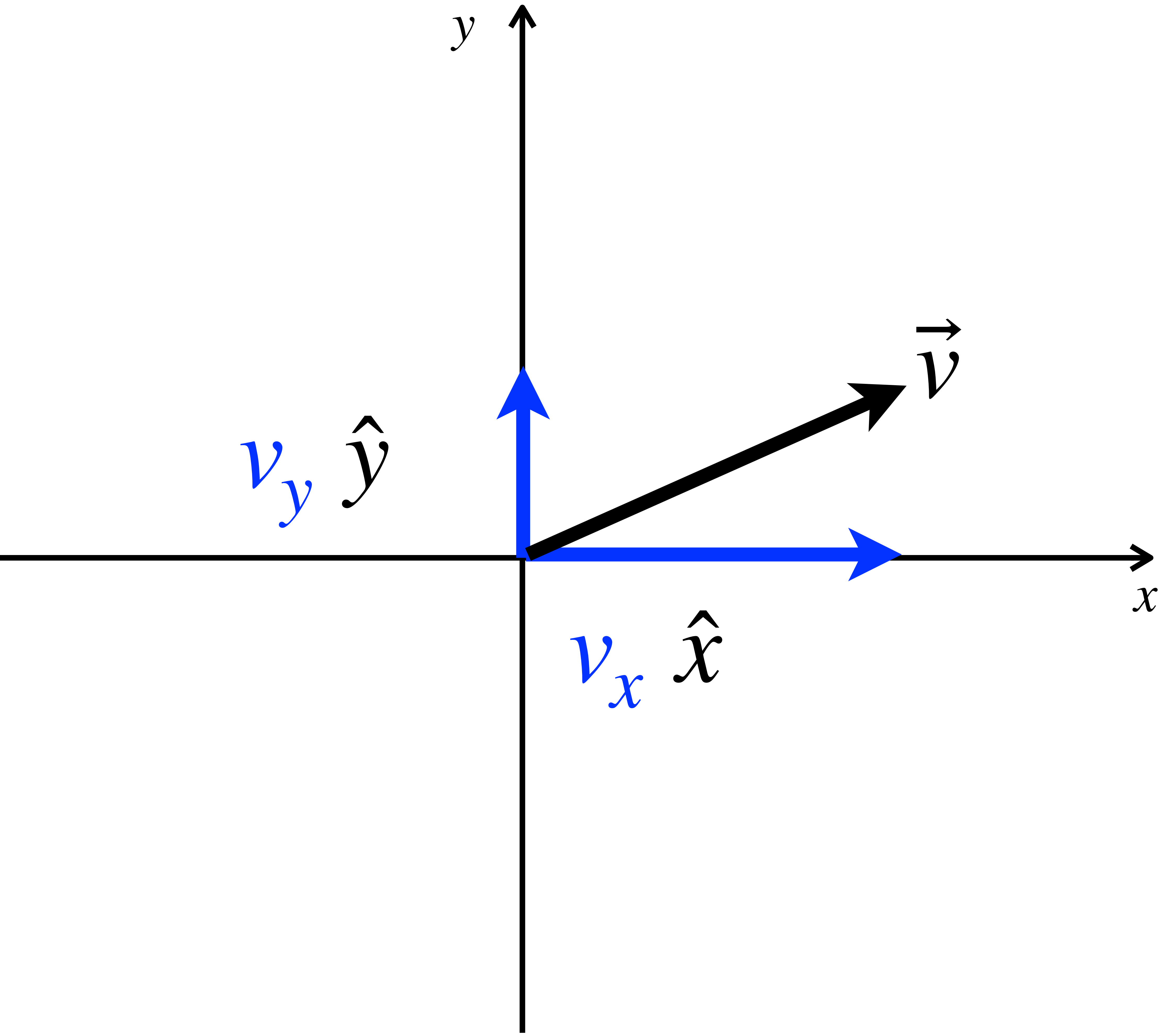
Solorio & Gire, in Review

A Special Form of 1:
Completeness Relations

Geometry of Completeness Relations

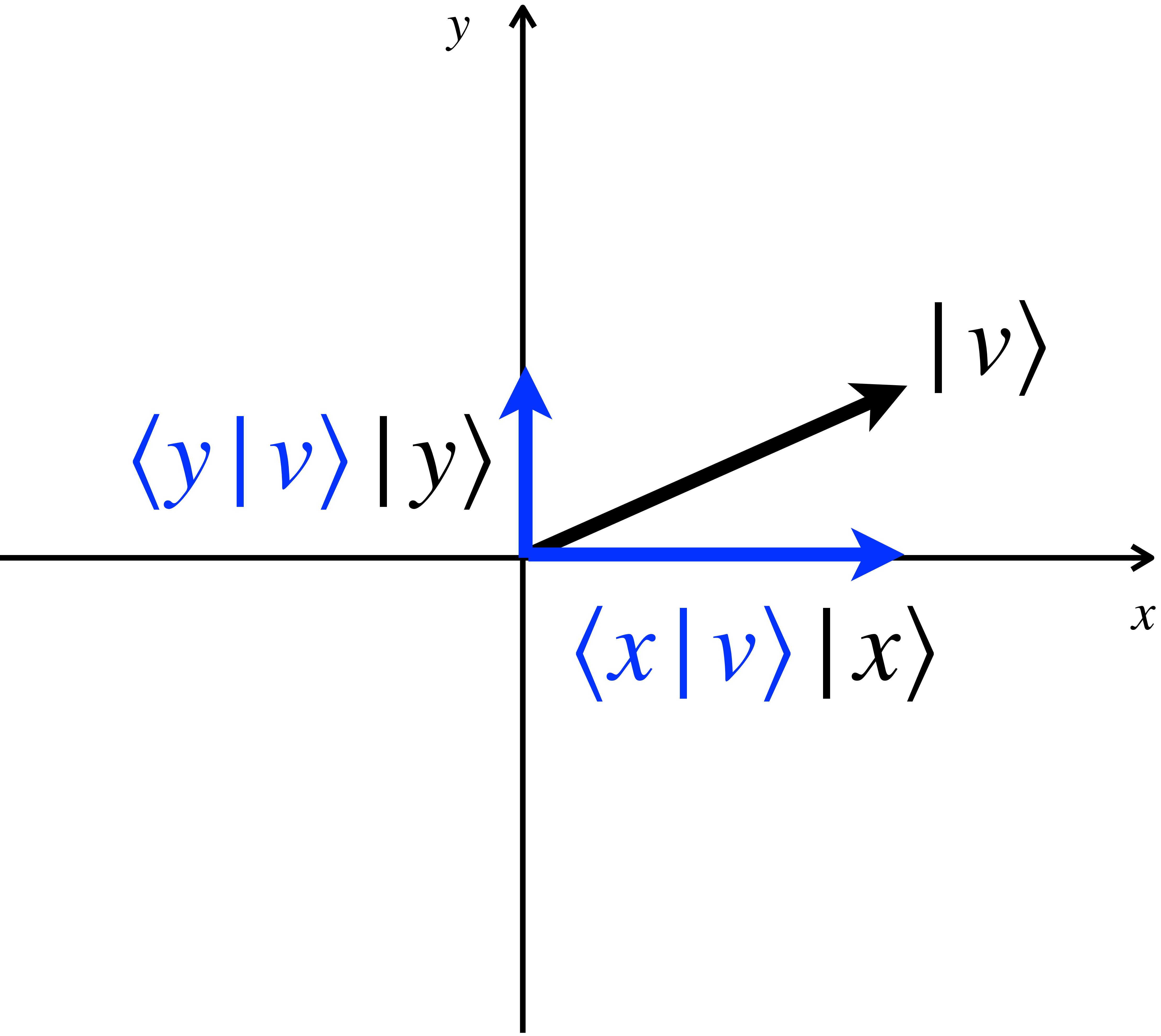


Geometry of Completeness Relations



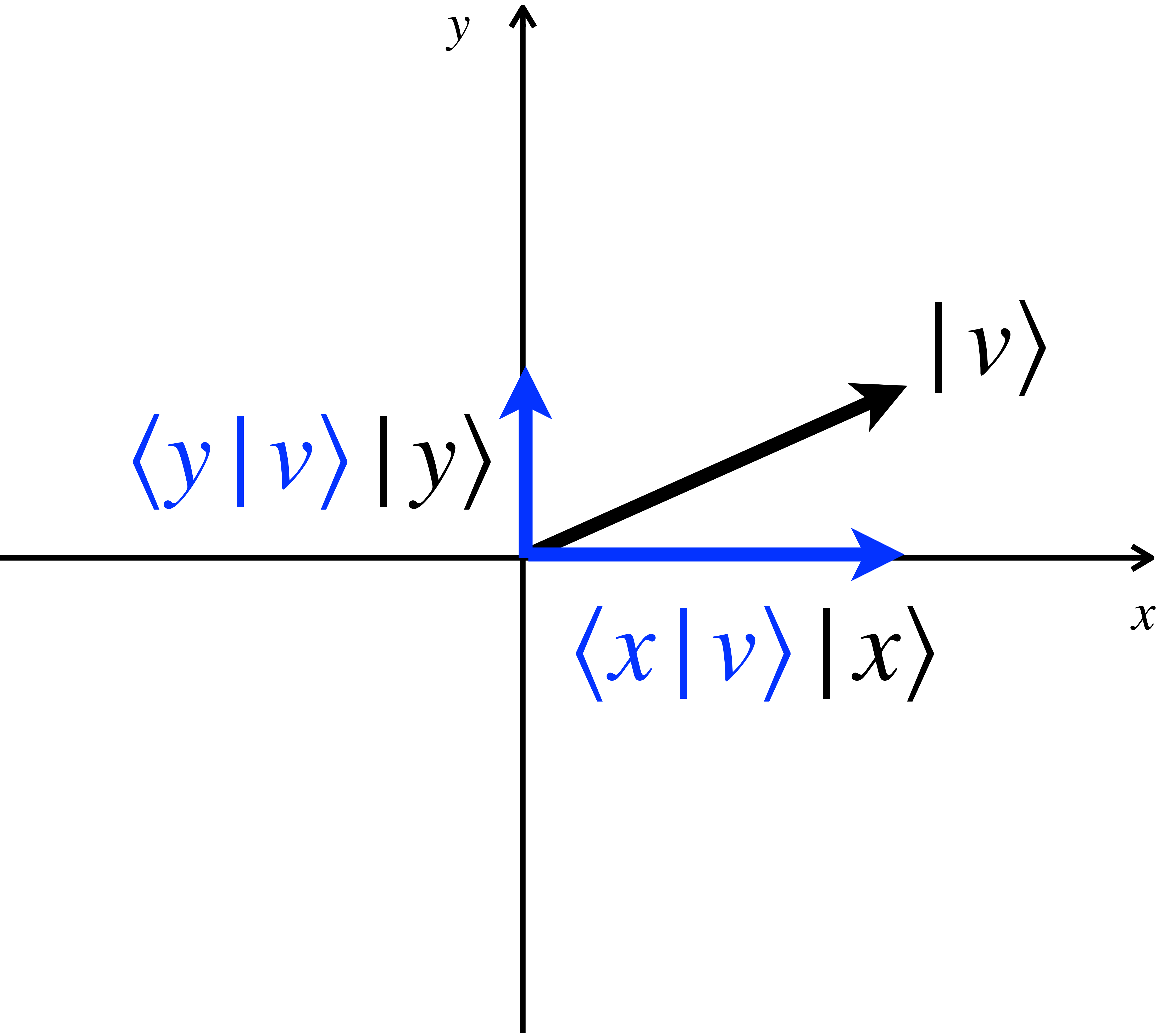
$$\vec{v} = v_x\,\hat{x} + v_y\,\hat{y}$$

Geometry of Completeness Relations



$$\vec{v} = v_x\,\hat{x} + v_y\,\hat{y}$$

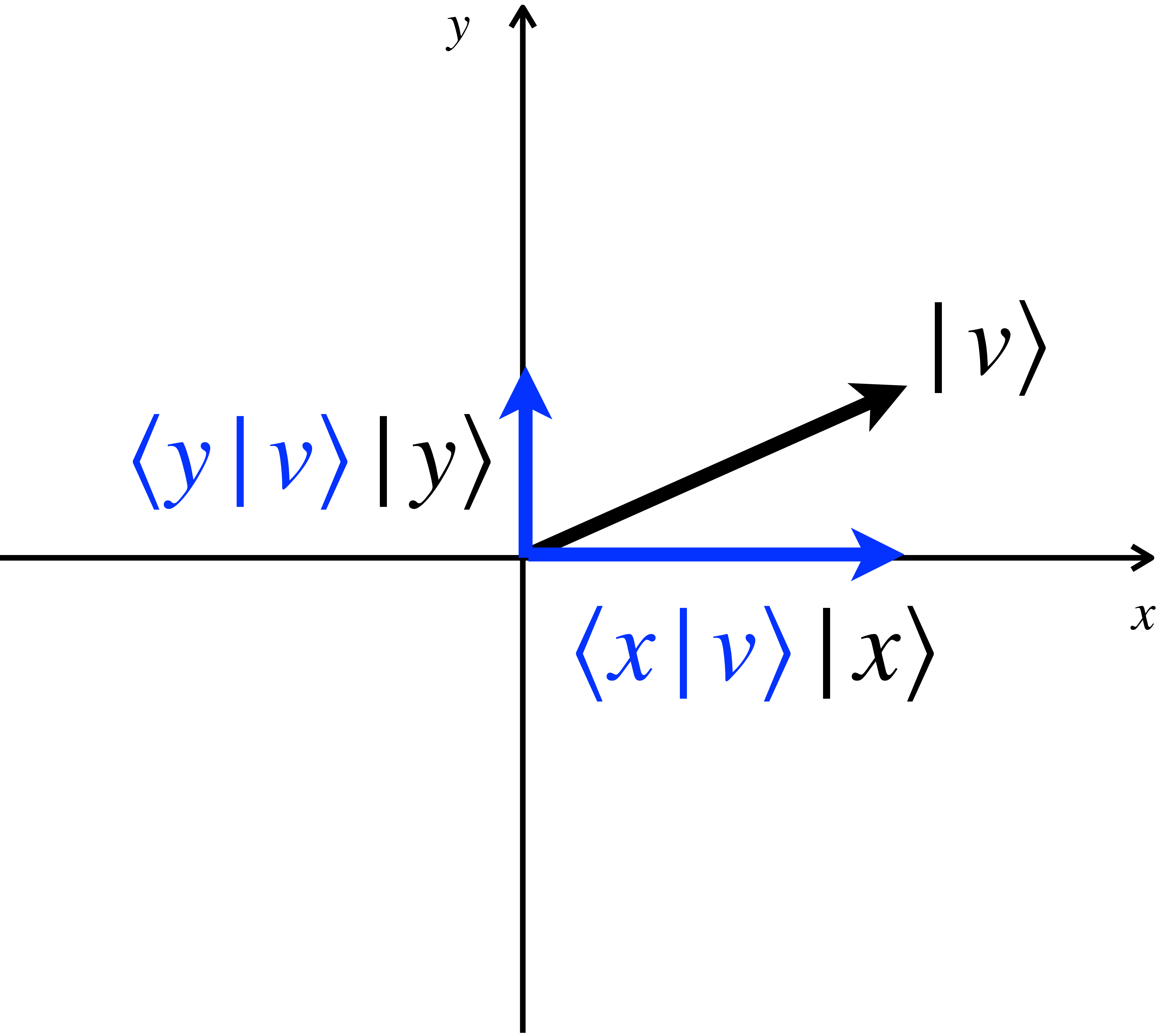
Geometry of Completeness Relations



$$\vec{v} = v_x\,\hat{x} + v_y\,\hat{y}$$

$$|v\rangle = \langle x|v\rangle\,|x\rangle + \langle y|v\rangle\,|y\rangle$$

Geometry of Completeness Relations

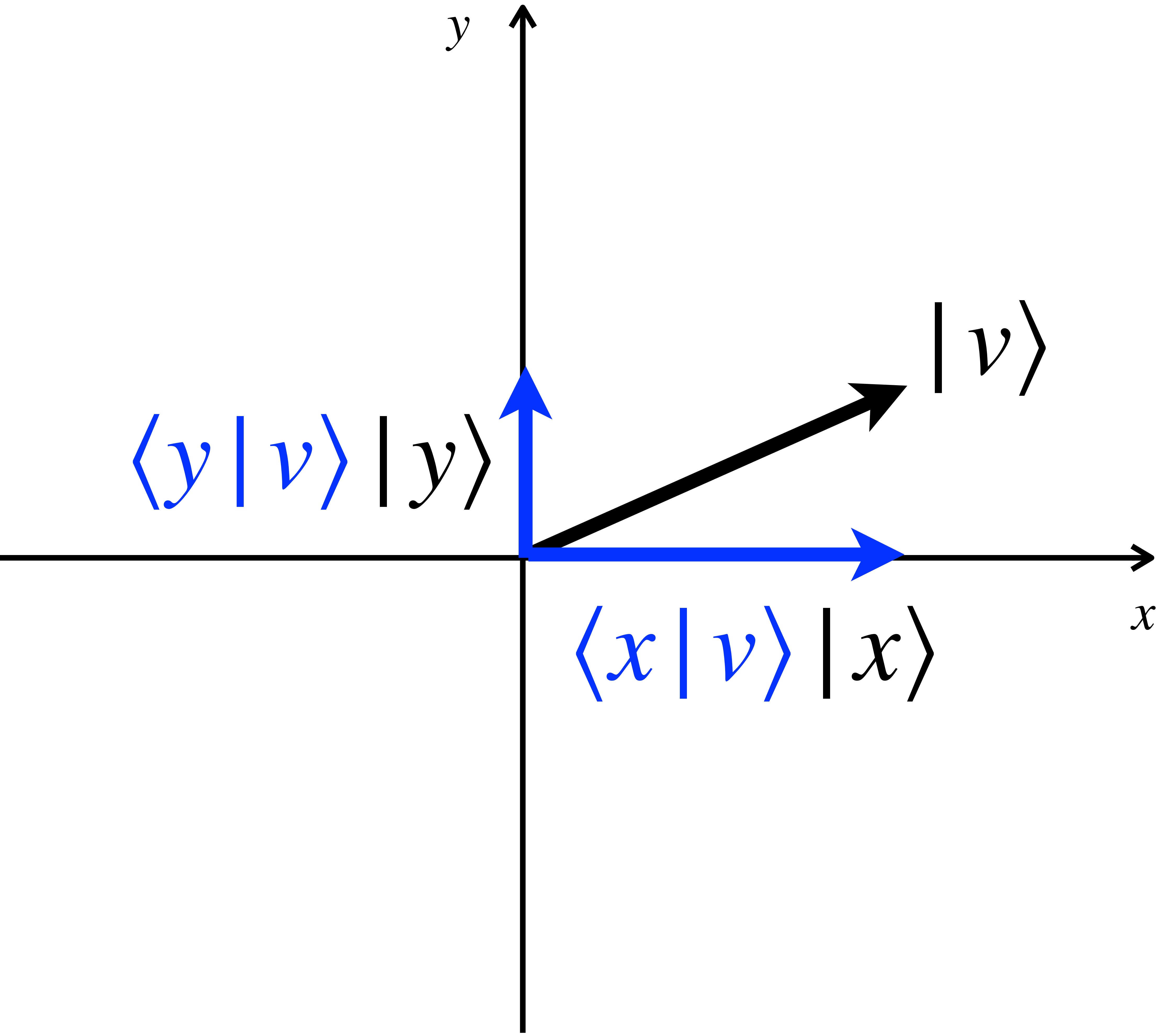


$$\vec{v} = v_x\,\hat{x} + v_y\,\hat{y}$$

$$|v\rangle = \langle x|v\rangle\,|x\rangle + \langle y|v\rangle\,|y\rangle$$

$$= |x\rangle\,\langle x|v\rangle + |y\rangle\,\langle y|v\rangle$$

Geometry of Completeness Relations



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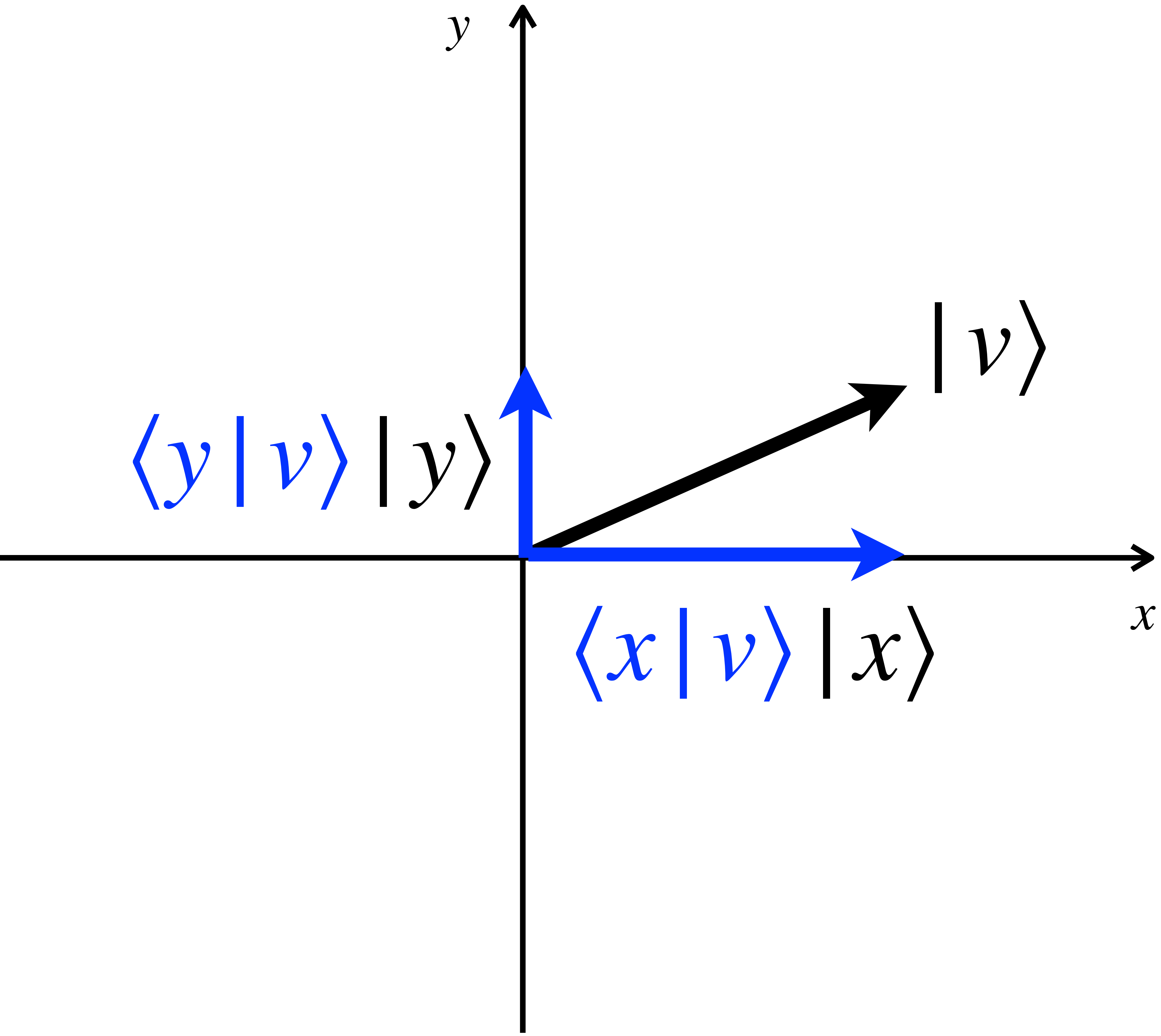
Geometry of Completeness Relations



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$$= |x\rangle\langle x| \, |v\rangle + |y\rangle\langle y| \, |v\rangle$$

$$|v\rangle = \Big(|x\rangle\langle x| + |y\rangle\langle y| \Big) \, |v\rangle$$

Completeness (Closure) Relations

Spin ½ System $\quad |+\rangle_{zz}\langle+| + |-\rangle_{zz}\langle-| = 1$

Infinite Square Well $\quad \sum_{n=1}^{\infty} |n\rangle\langle n| = 1 \qquad \int_0^L |x\rangle\langle x|\,dx = 1$

Dimensionless

length

Dimensionless

Dimensionless Dimensionless

$\frac{1}{\sqrt{length}}$ $\frac{1}{\sqrt{length}}$

Writing a State in a Basis

$$|\psi\rangle = (1)|\psi\rangle$$

$$= \left(\sum_n |n\rangle\langle n|\right)|\psi\rangle$$

$$= \sum_n |n\rangle\langle n|\psi\rangle$$

$$= \sum_n \langle n|\psi\rangle|n\rangle$$

$$= \sum_n c_n|n\rangle$$

$$|\psi\rangle \doteq c_n$$

"Probability amplitude"

$$|\psi\rangle = (1)|\psi\rangle$$

$$= \left(\int |x\rangle\langle x|\,dx\right)|\psi\rangle$$

Distribute

$$= \int |x\rangle\langle x|\psi\rangle\,dx$$

Rearrange

$$= \int \langle x|\psi\rangle|x\rangle\,dx$$

Interpret

$$= \int \psi(x)|x\rangle dx$$

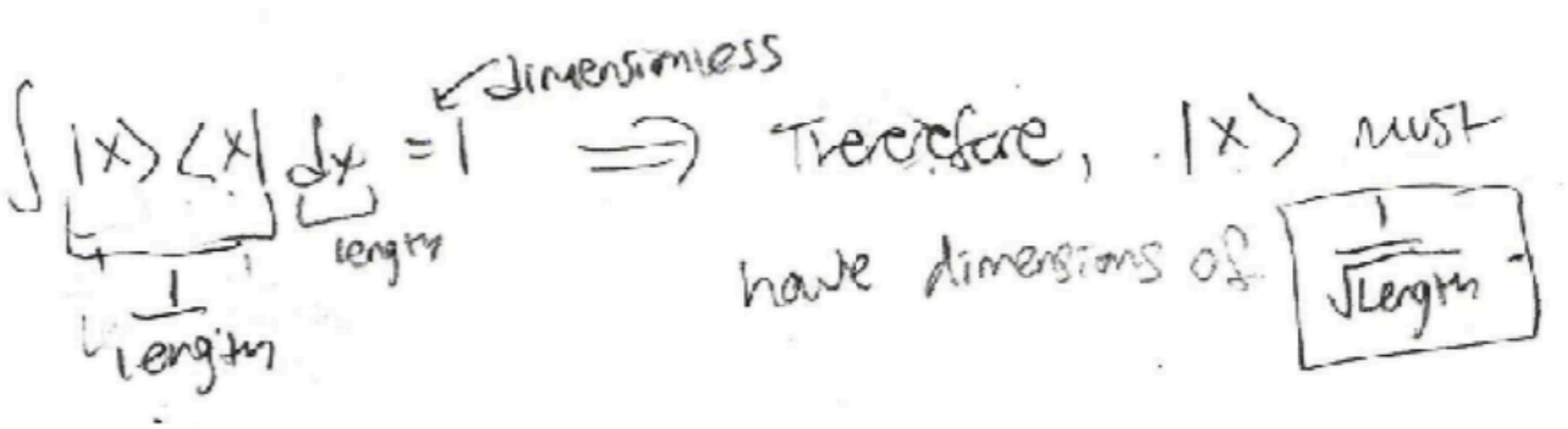
Short-hand

$$|\psi\rangle \doteq \psi(x)$$

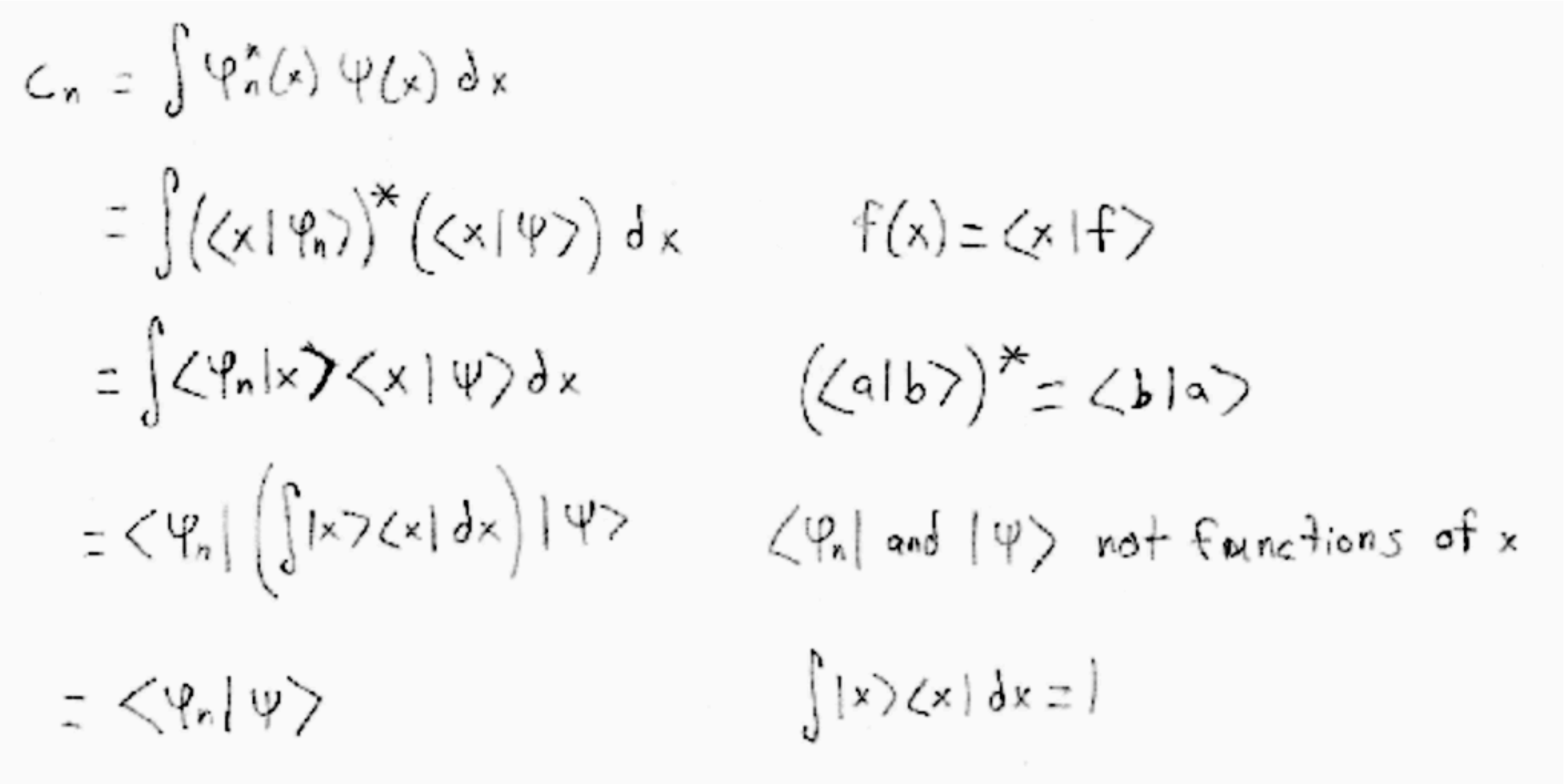
"Probability density amplitude" "Wavefunction"

PER around Completeness Relations

- Physical dimensions of kets

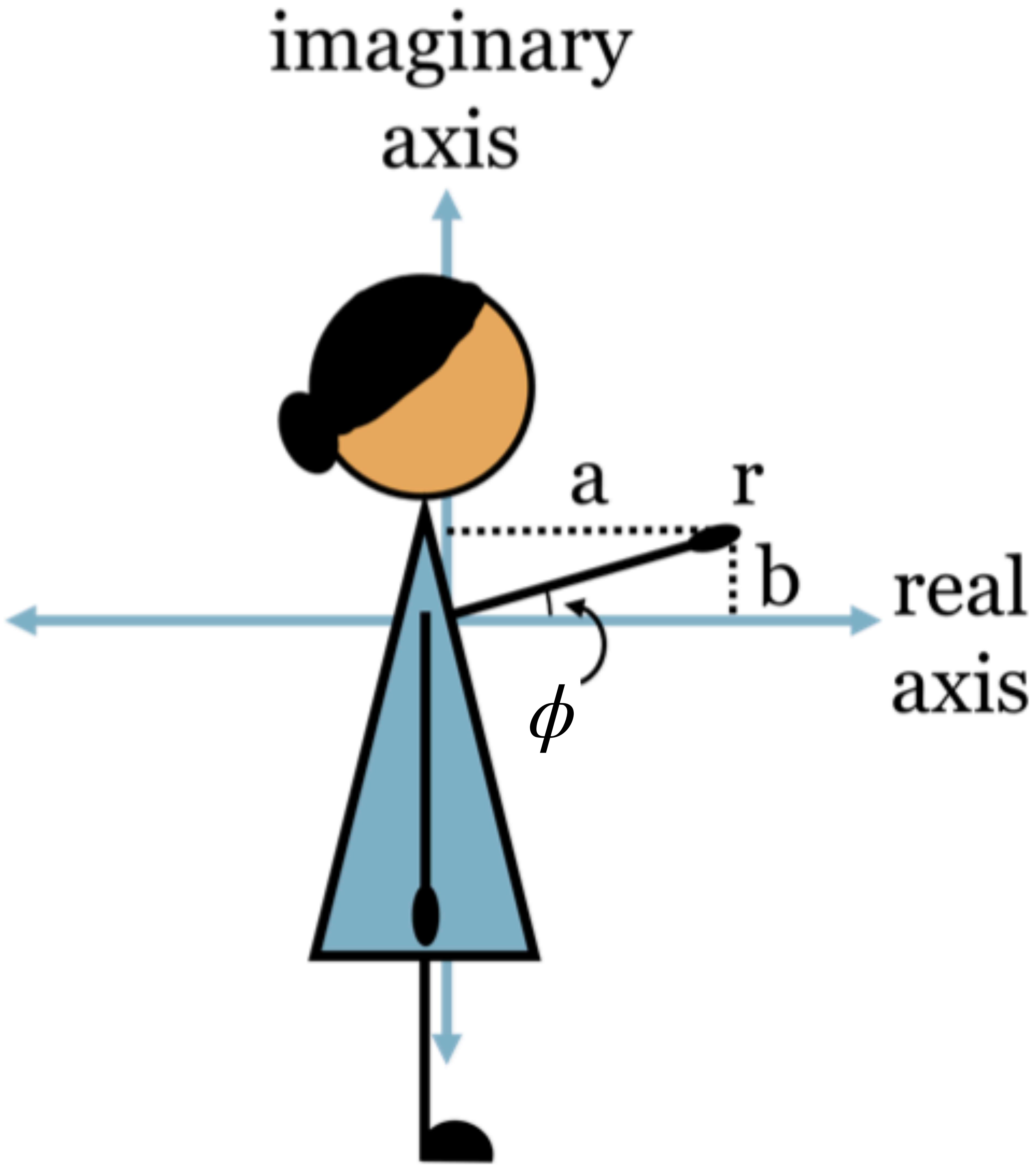


- Translating between wavefunctions & kets



Arms Representation

Arms Basics

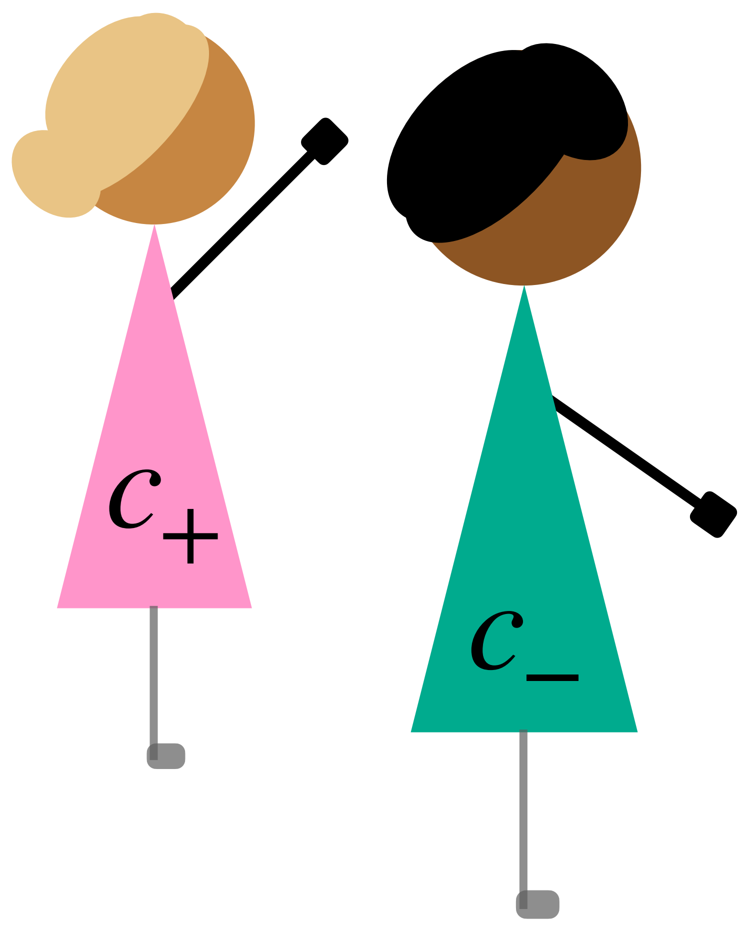


Quantum Concepts & Arms

Quantum states are vectors with complex components

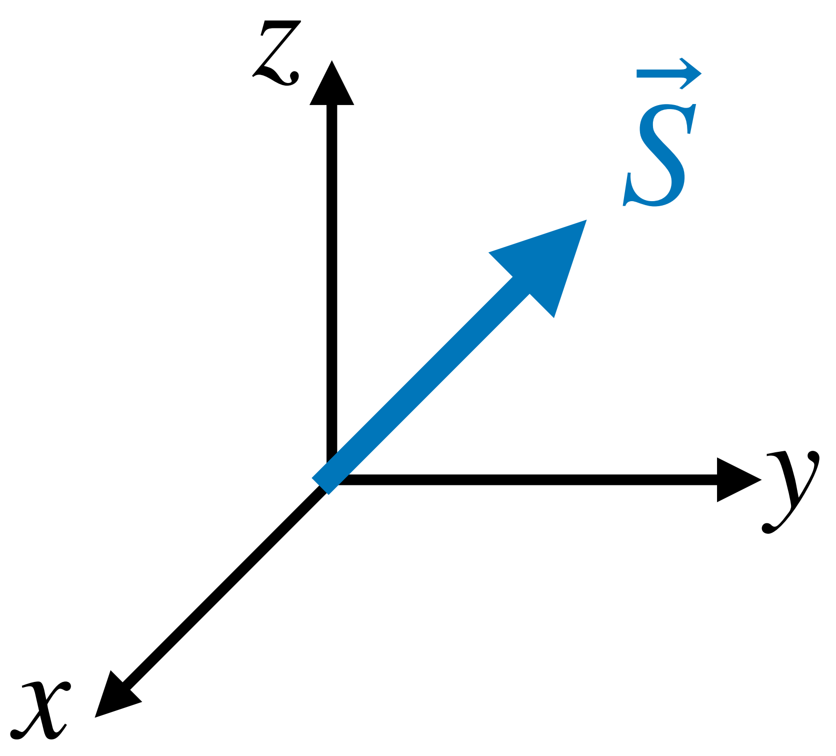
$$|\psi\rangle = c_+ |+\rangle_z + c_- |-\rangle_z \qquad |\psi\rangle \doteq \begin{bmatrix} c_+ \\ c_- \end{bmatrix}$$

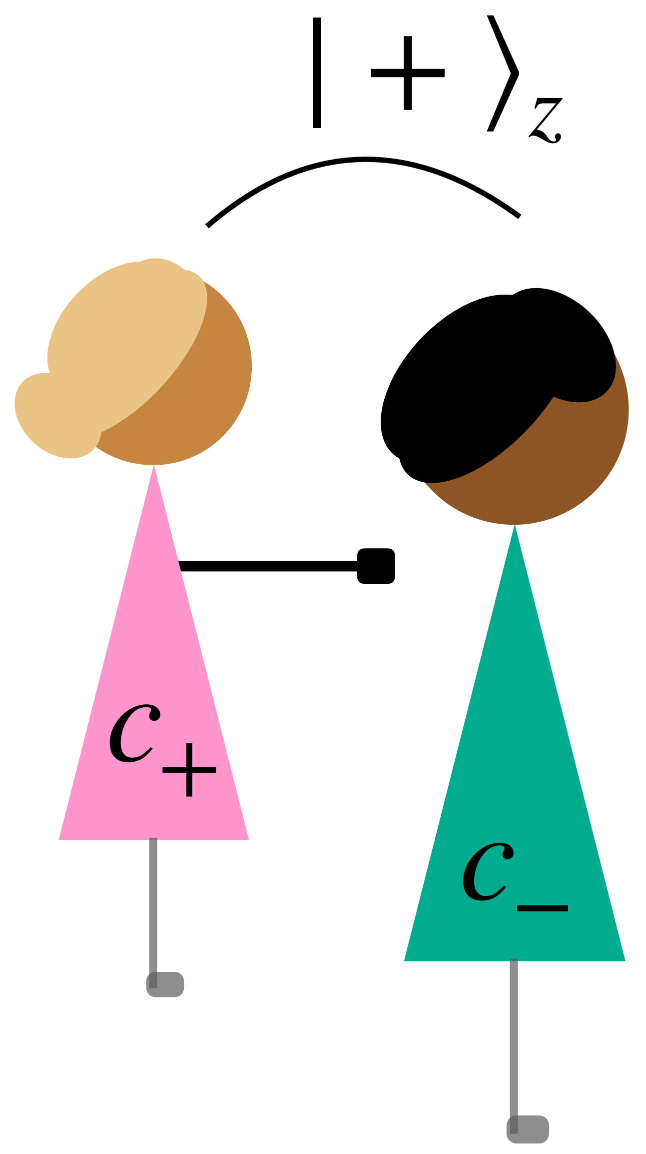
$${}_z\langle + | \psi \rangle \qquad {}_z\langle - | \psi \rangle$$



Quantum Concepts & Arms

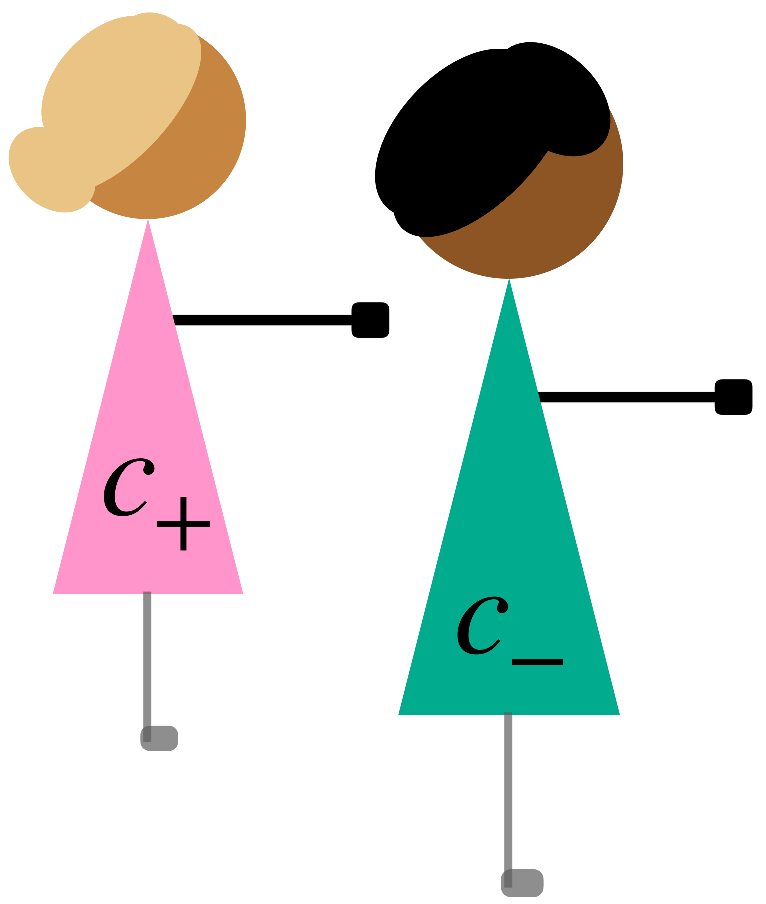
Cartesian space and Hilbert space are different



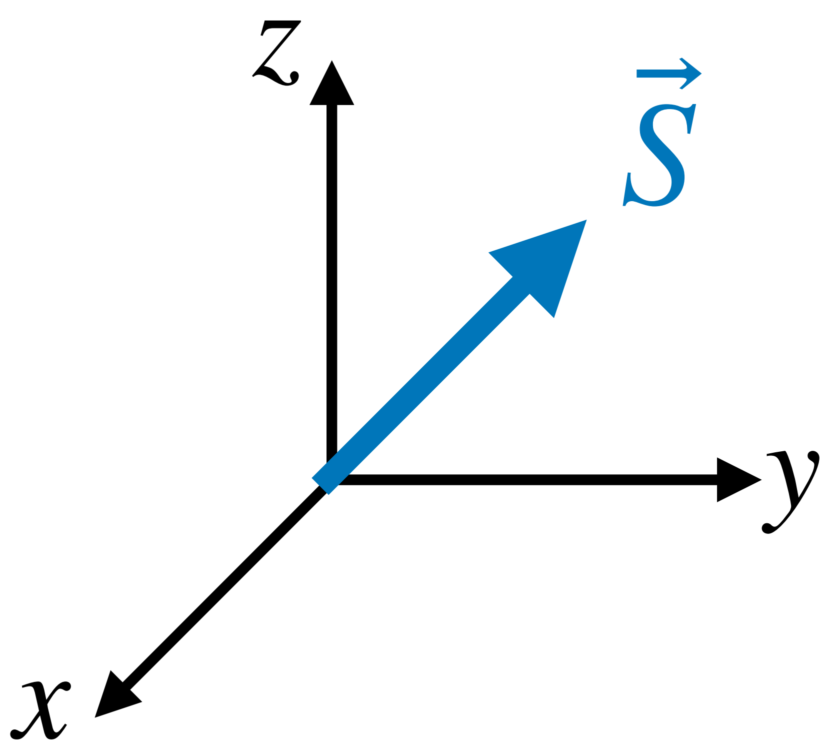


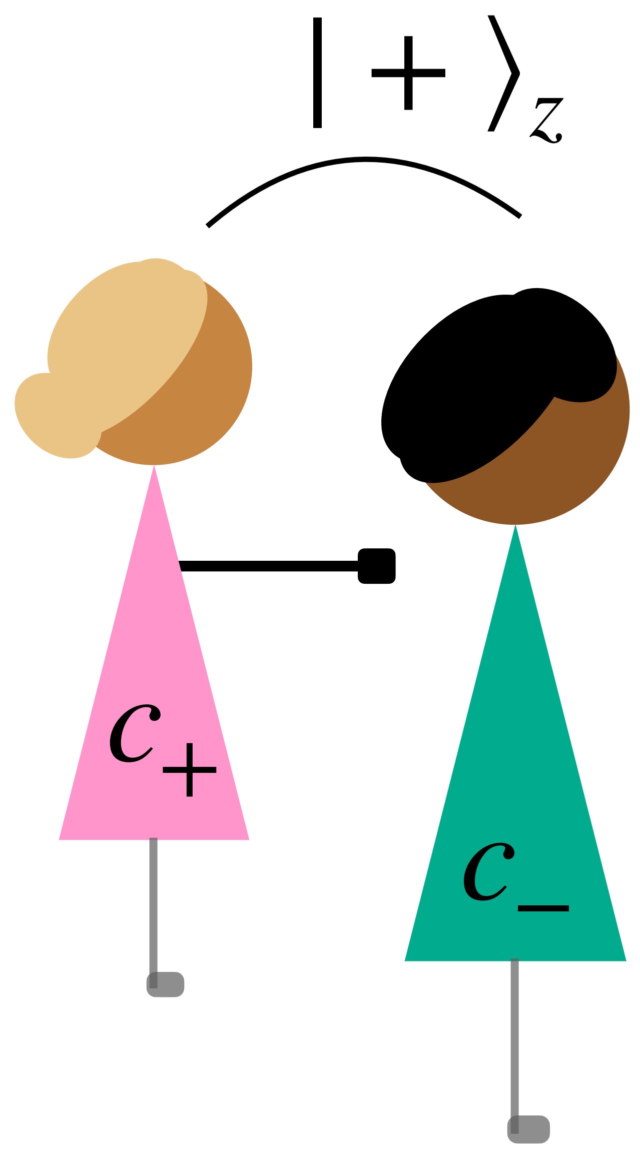
Quantum Concepts & Arms

$$|+\rangle_x = \frac{1}{\sqrt{2}}|+\rangle_z + \frac{1}{\sqrt{2}}|-\rangle_z$$



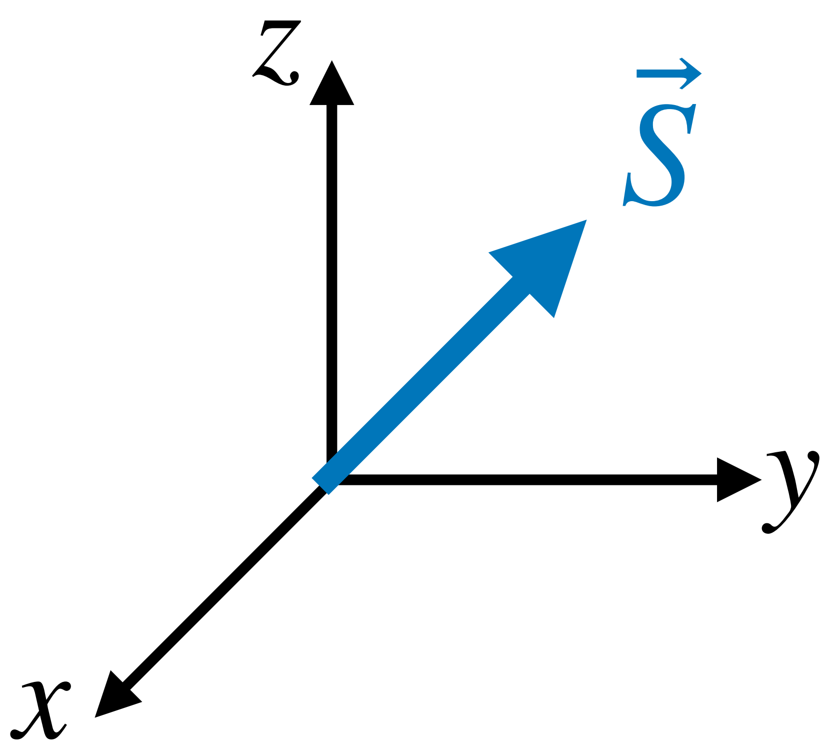
Cartesian space and Hilbert space are different

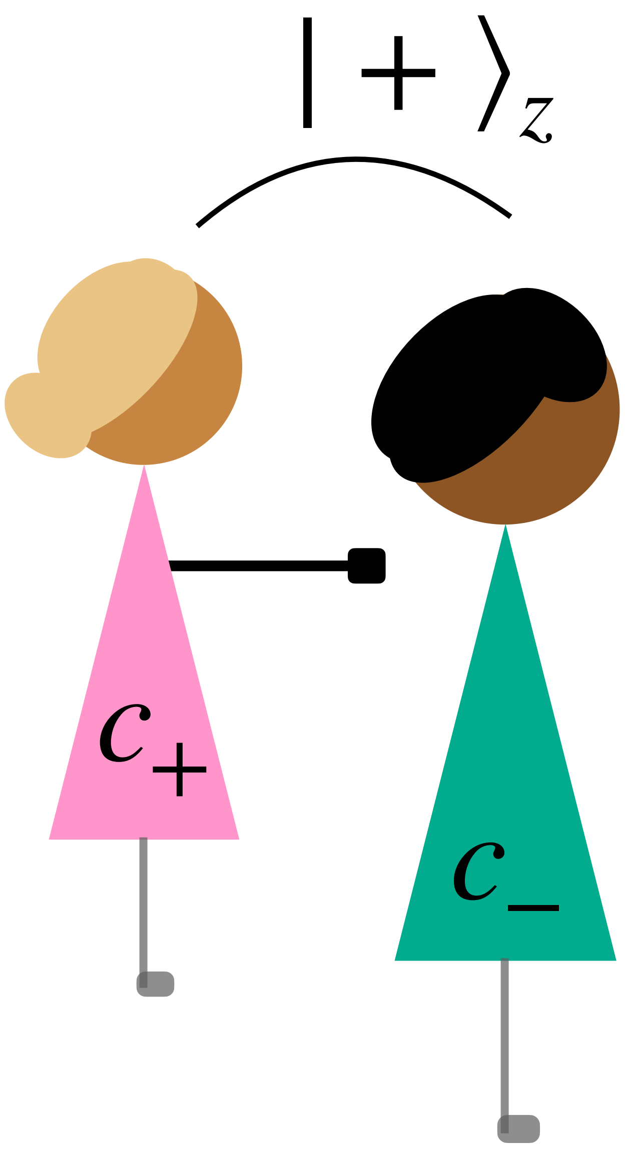




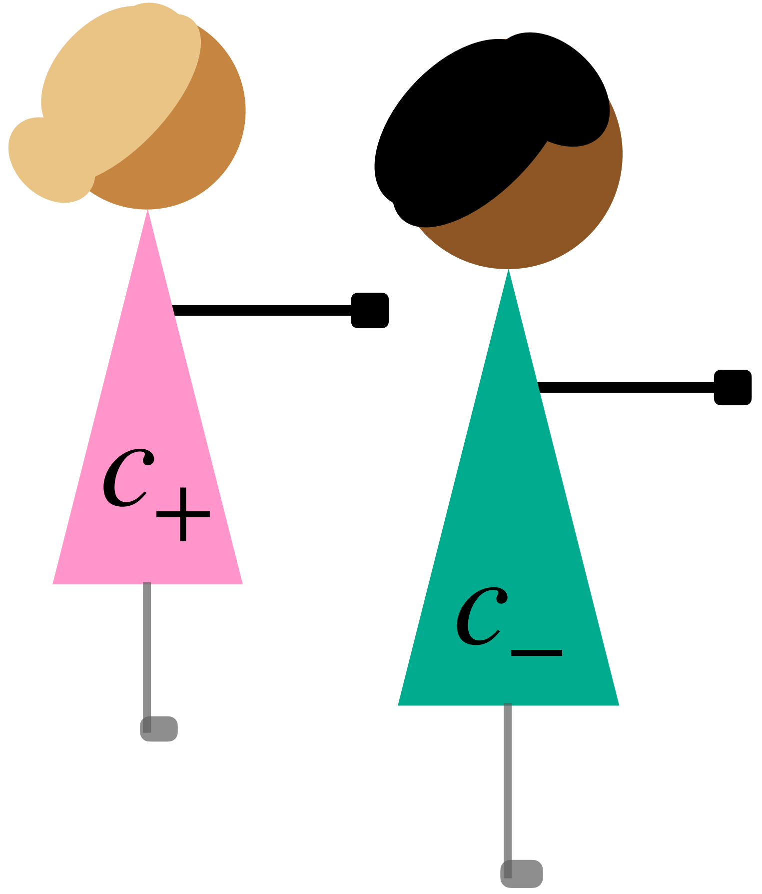
Quantum Concepts & Arms

Cartesian space and Hilbert space are different

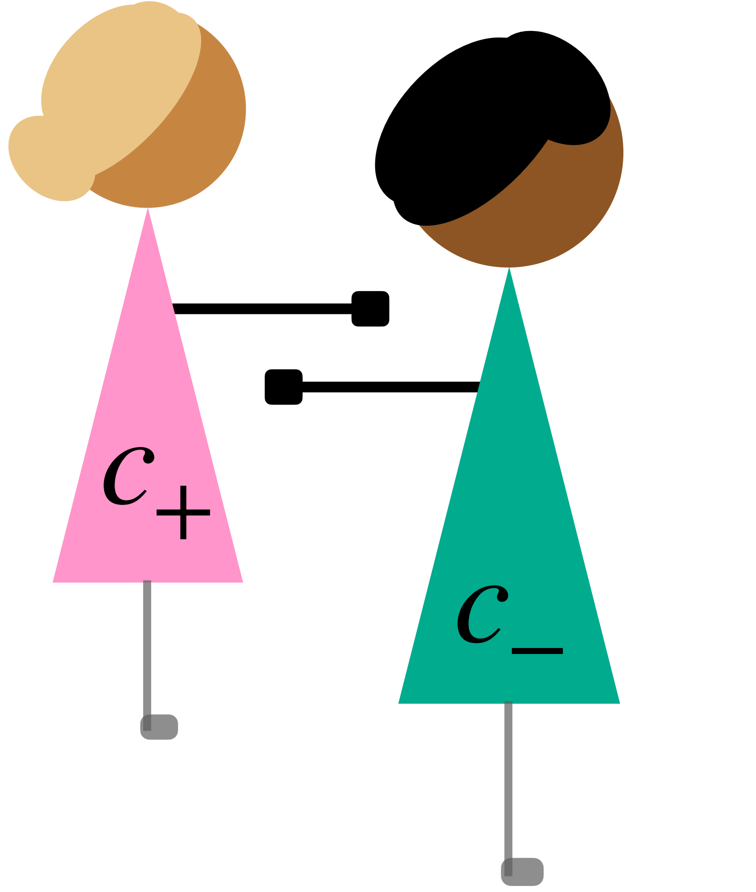




$$|+\rangle_x = \frac{1}{\sqrt{2}}|+\rangle_z + \frac{1}{\sqrt{2}}|-\rangle_z$$



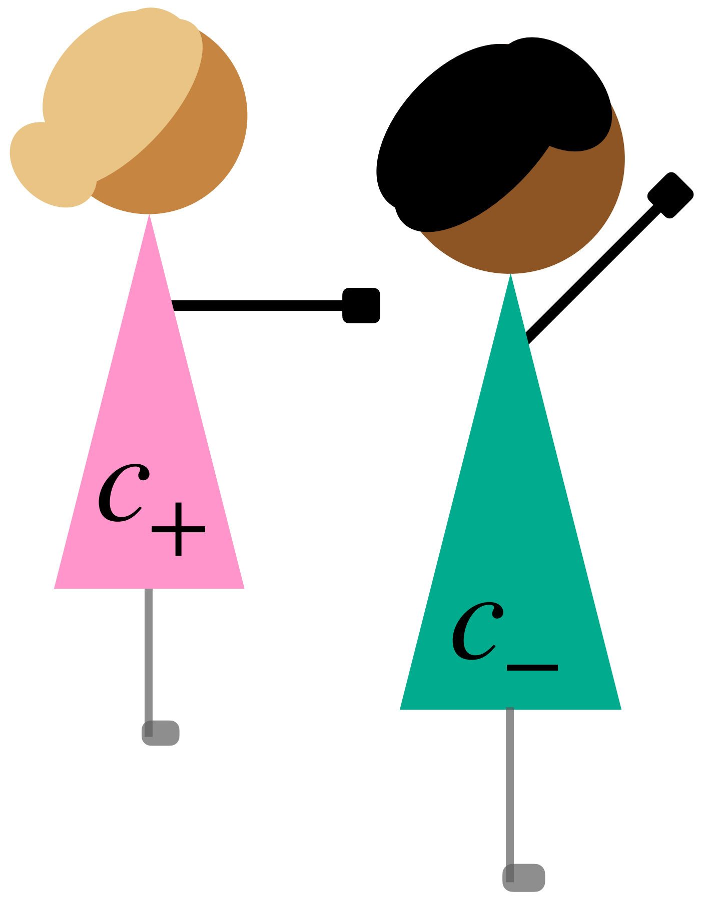
$$|-\rangle_x = \frac{1}{\sqrt{2}}|+\rangle_z - \frac{1}{\sqrt{2}}|-\rangle_z$$

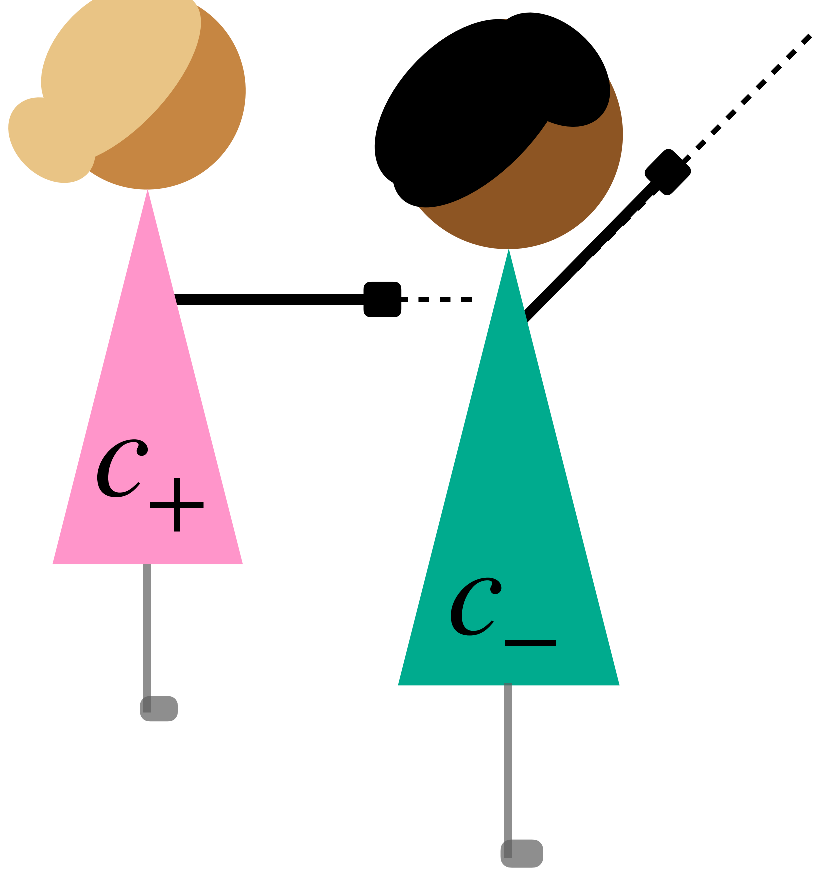


Quantum Concepts & Arms

Vectors that differ by an overall phase represent the same quantum state

$$|\psi\rangle = c_+|+\rangle + c_-|-\rangle \qquad\qquad |\psi\rangle = e^{i\phi}\Big(c_+|+\rangle + c_-|-\rangle\Big)$$

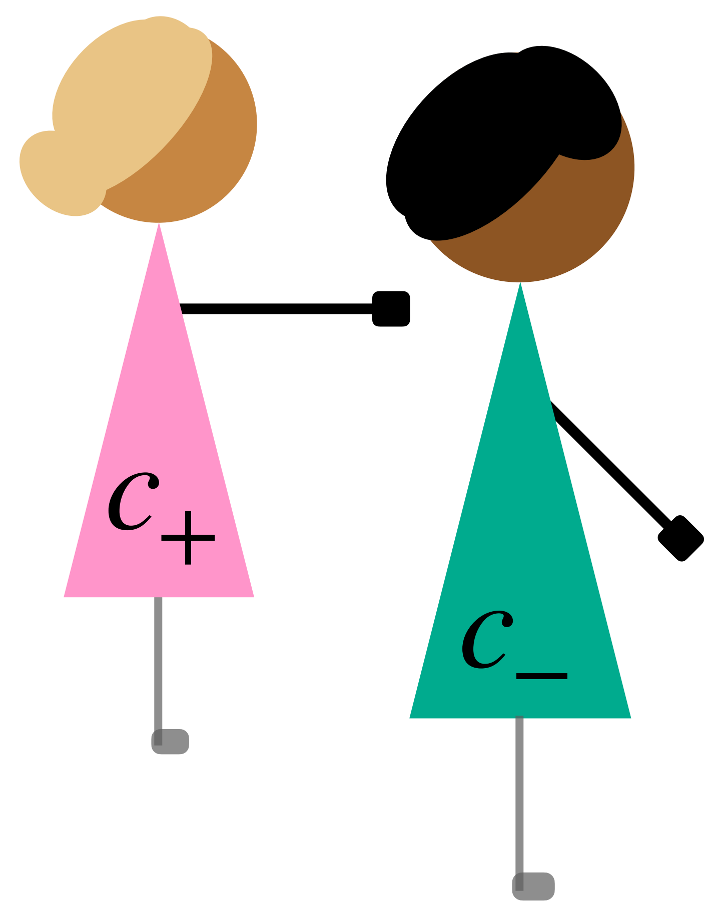




Quantum Concepts & Arms

Quantum states evolve with time - time & energy-dependent phase on terms in energy eigenstate expansion

$$|\psi(t)\rangle = c_+ e^{-iE_+ t/\hbar} |+\rangle + c_- e^{-iE_- t/\hbar} |-\rangle$$



Wavefunction with Arms

Formalisms for discrete and continuous quantum systems are related.

$$c_{\pm} = {}_{z}\langle \pm | \psi \rangle \qquad \psi(x) = \langle x | \psi \rangle$$

Kinesthetic Activities for *Upper Division Quantum Mechanics?!*

Activate sensorimotor brain systems

Make decisions about how configure and move sequentially

Re-representation

For quantum systems (>1 people), have to socially negotiate

Introduces silliness and laughter

Formative assessment

Hahn & Gire, *Am. J. Phys.*, 2022
Solomon, et al., *Phys. Ed.*, 1991
Kontra, et al., *Psychol. Sci.*, 2015
Duijzer, et al., Educ. *Psychol. Rev.*, 2019
Struck & Yerrick, *J. Sci Educ. Technol.*, 2010,
Beichner, et al., *Am. J. Phys.*, 1990
Hubber, Titler, & Haslam, *Res. Sci. Educ.*, 2010

Arms Affordances & Constraints

- ✓ 4D
- ✓ Phase Angle Salient
- ✓ Accommodate Physical Ability
- ✓ Components of complex numbers vs. quantum basis
- ✓ Memorable

Hahn & Gire, Am. J. Phys., 2022

Hahn, PhD Dissertation, 2022

Arms Affordances & Constraints

- ✓ 4D
- ✓ Phase Angle Salient
- ✓ Accommodate Physical Ability
- ✓ Components of complex numbers vs. quantum basis
- ✓ Memorable

- – Arm length not adjustable for different norms
- – Information that is not externalized
- – Visualization? (Literalness??)
- – Self Consciousness

Hahn & Gire, *Am. J. Phys.*, 2022

Hahn, PhD Dissertation, 2022

Arms Activities

Complex Numbers

Quantum State

Relative & Overall Phase

Time Evolution

Wavefunction

Hahn & Gire, *Am. J. Phys*, 2022

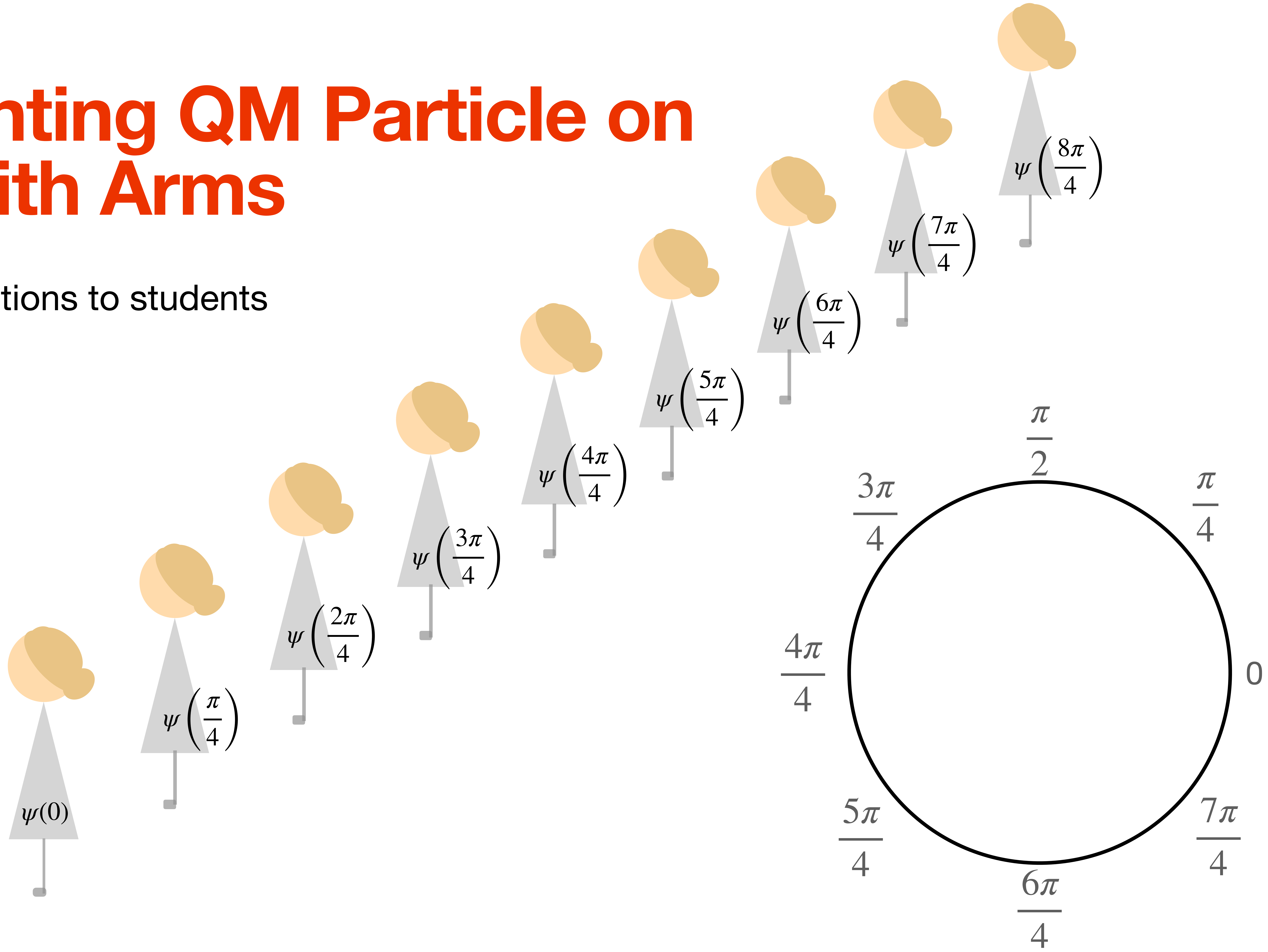
Inner Product of Spin-1/2 States

Time Evolution of a Particle on a Ring

New!

Representing QM Particle on a Ring with Arms

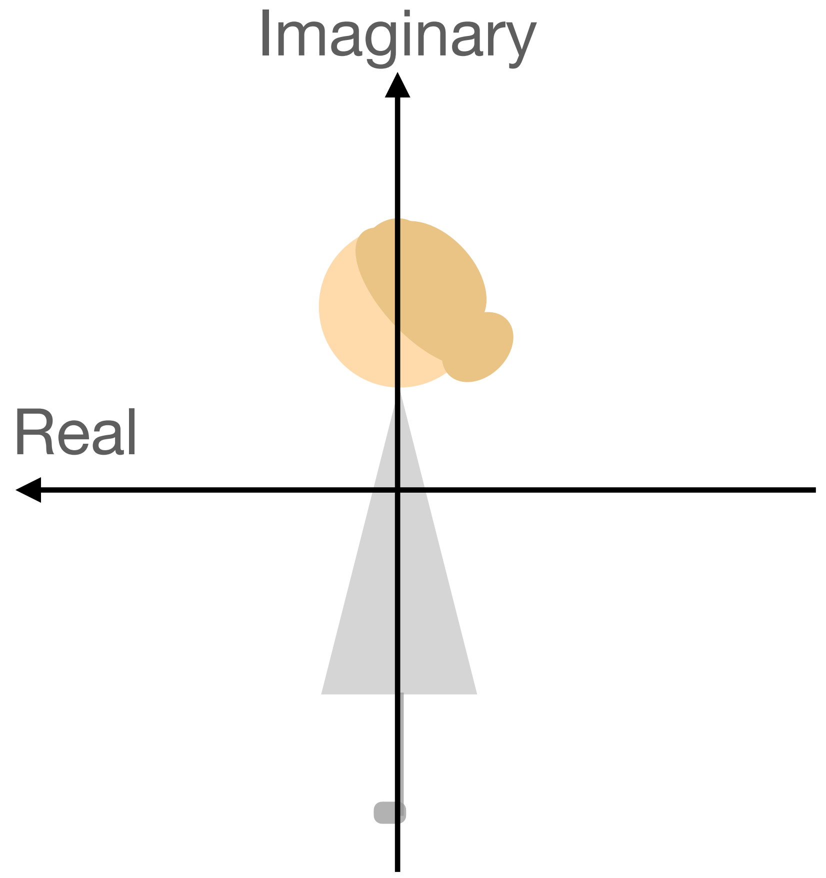
Assign angular positions to students

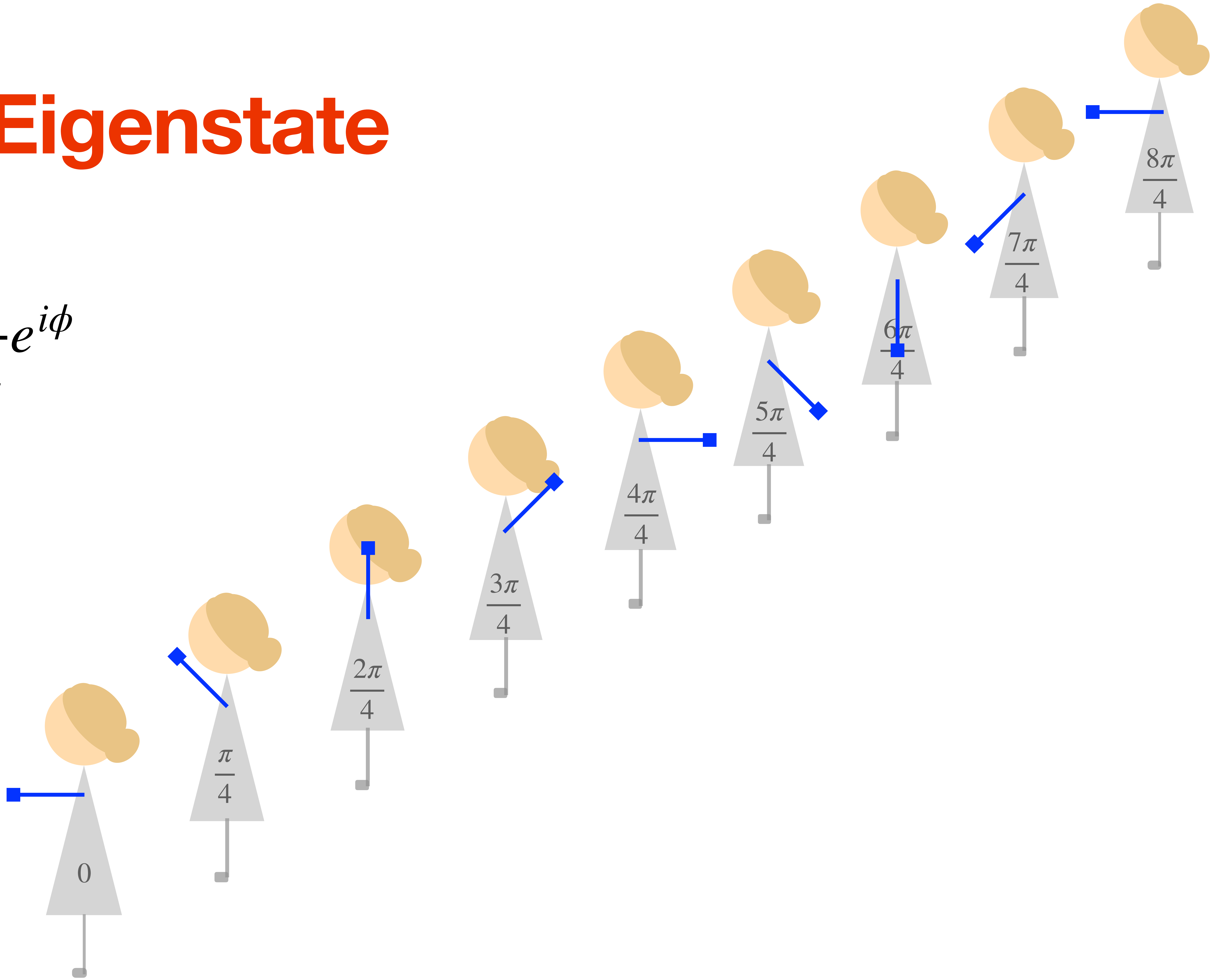


Energy Eigenstate

m=1

$$E_1(\phi) = \frac{1}{\sqrt{2\pi}} e^{i\phi}$$





Energy Eigenstate

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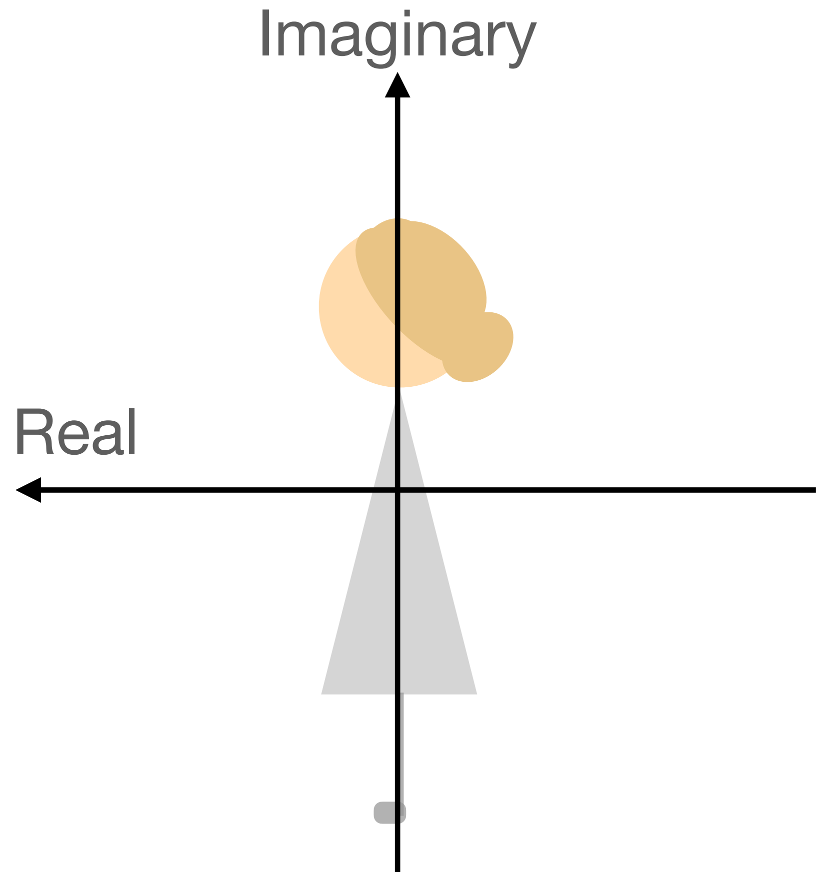
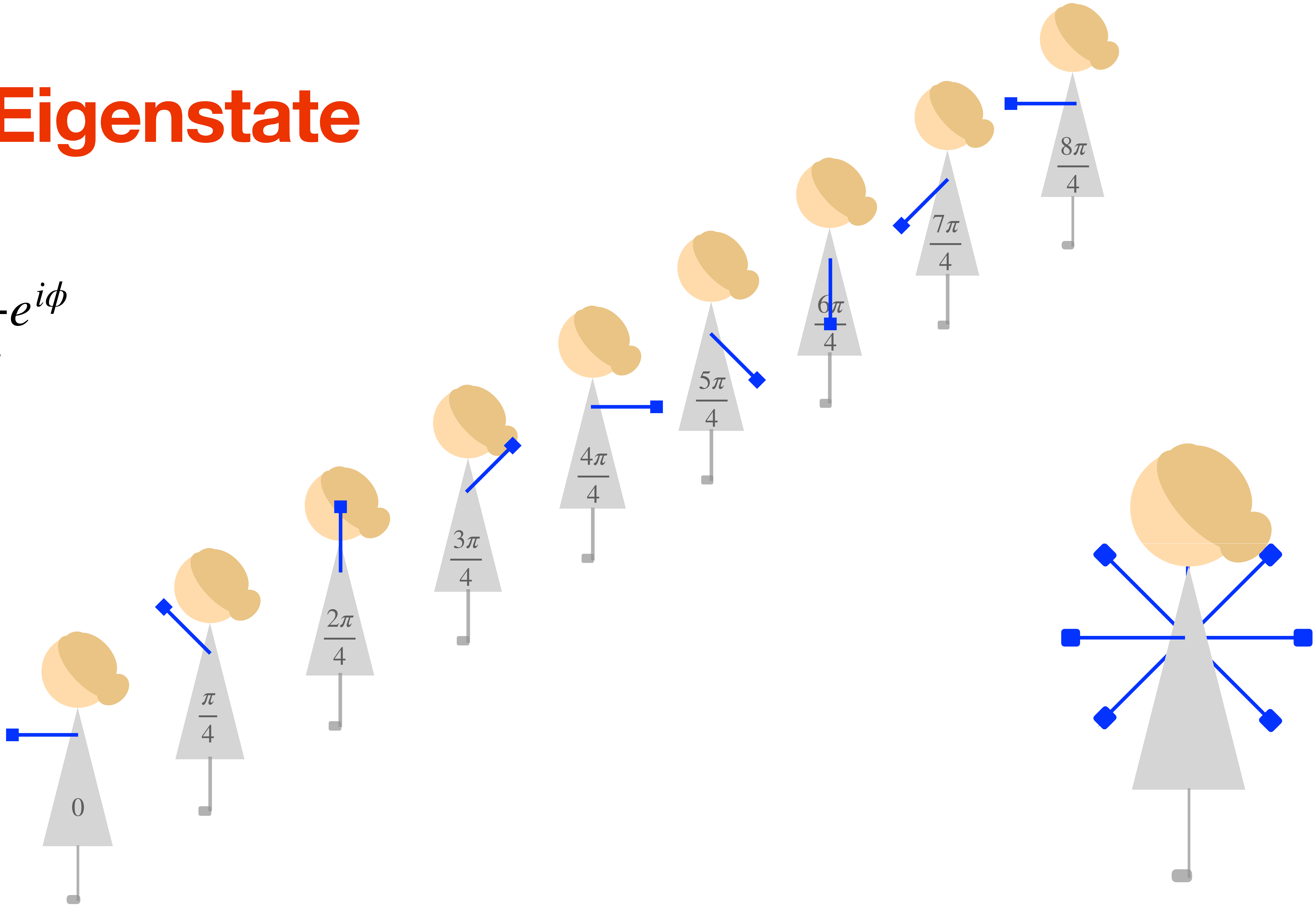


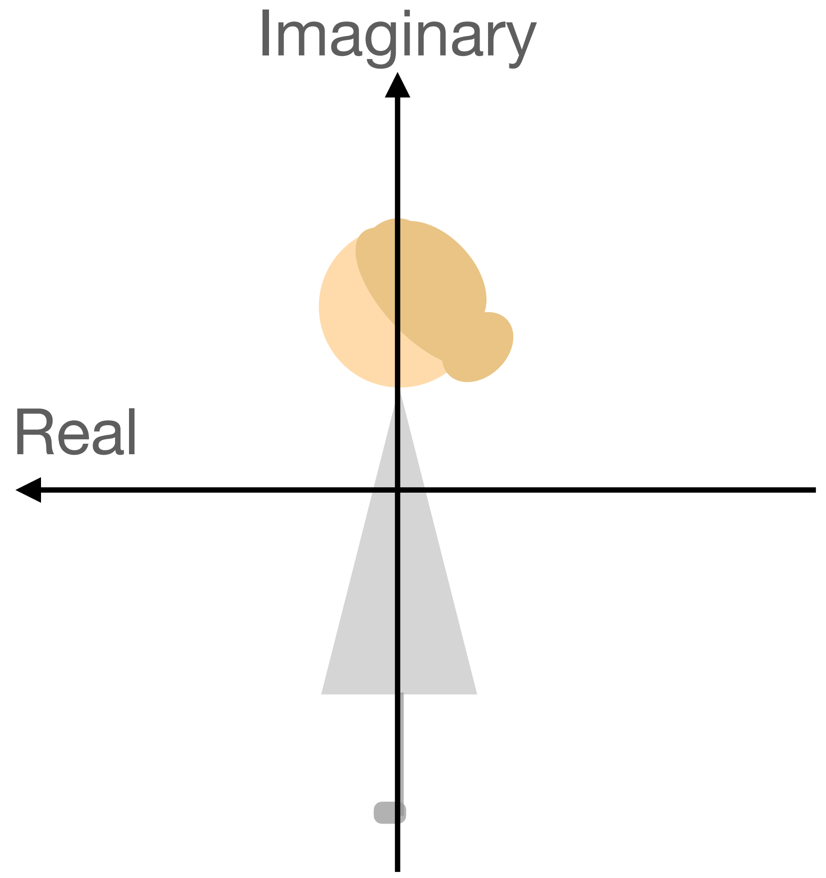


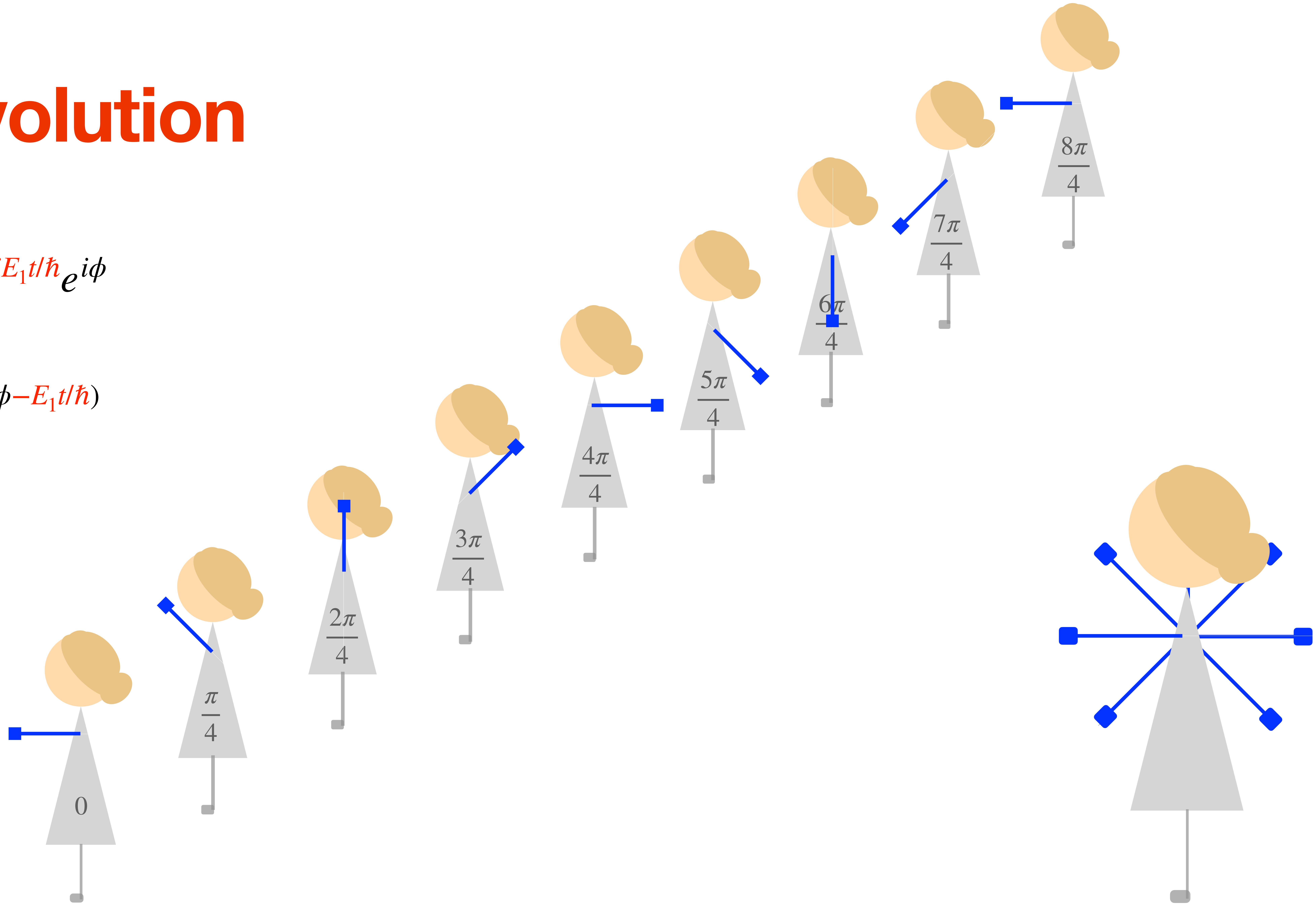
Time Evolution

m=1

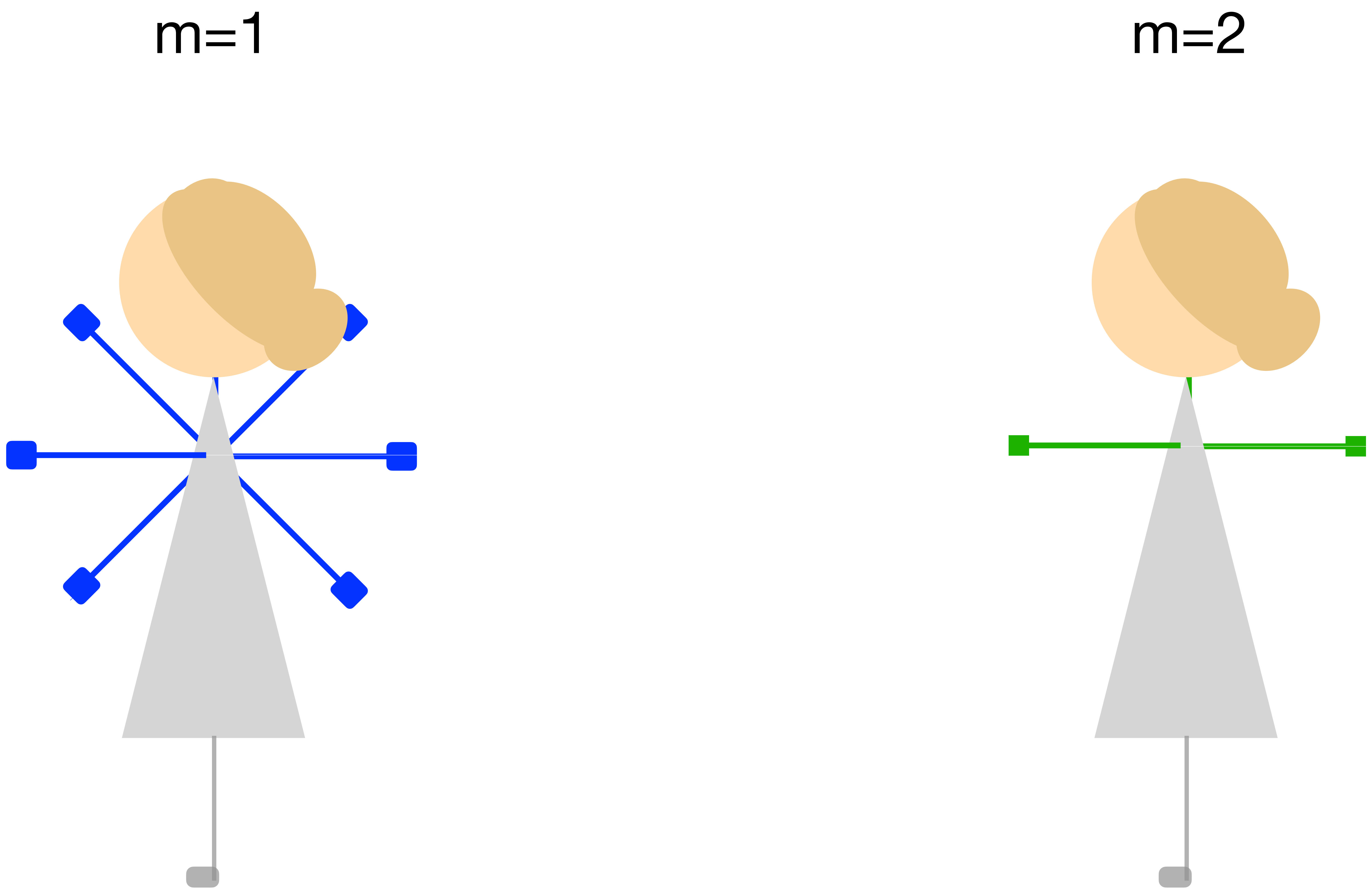
$$E_1(\phi) = \frac{1}{\sqrt{2\pi}} e^{-iE_1 t/\hbar} e^{i\phi}$$

$$= \frac{1}{\sqrt{2\pi}} e^{i(\phi - E_1 t/\hbar)}$$





Time Evolution

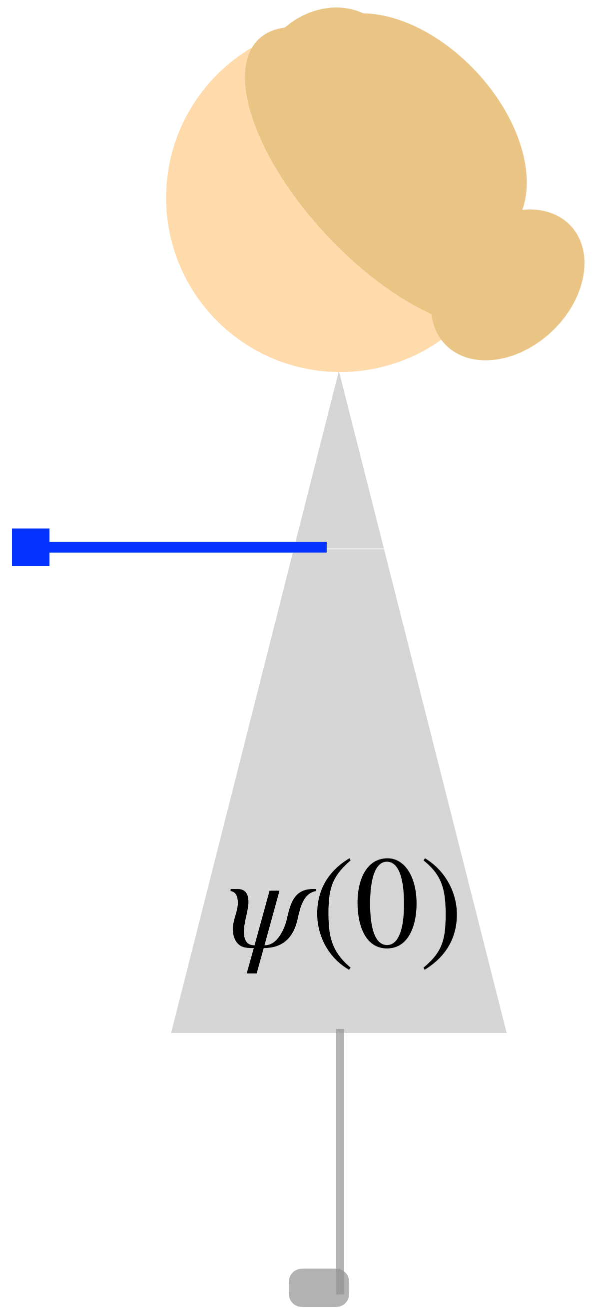


$$E_2 = 4E_1$$

Superposition

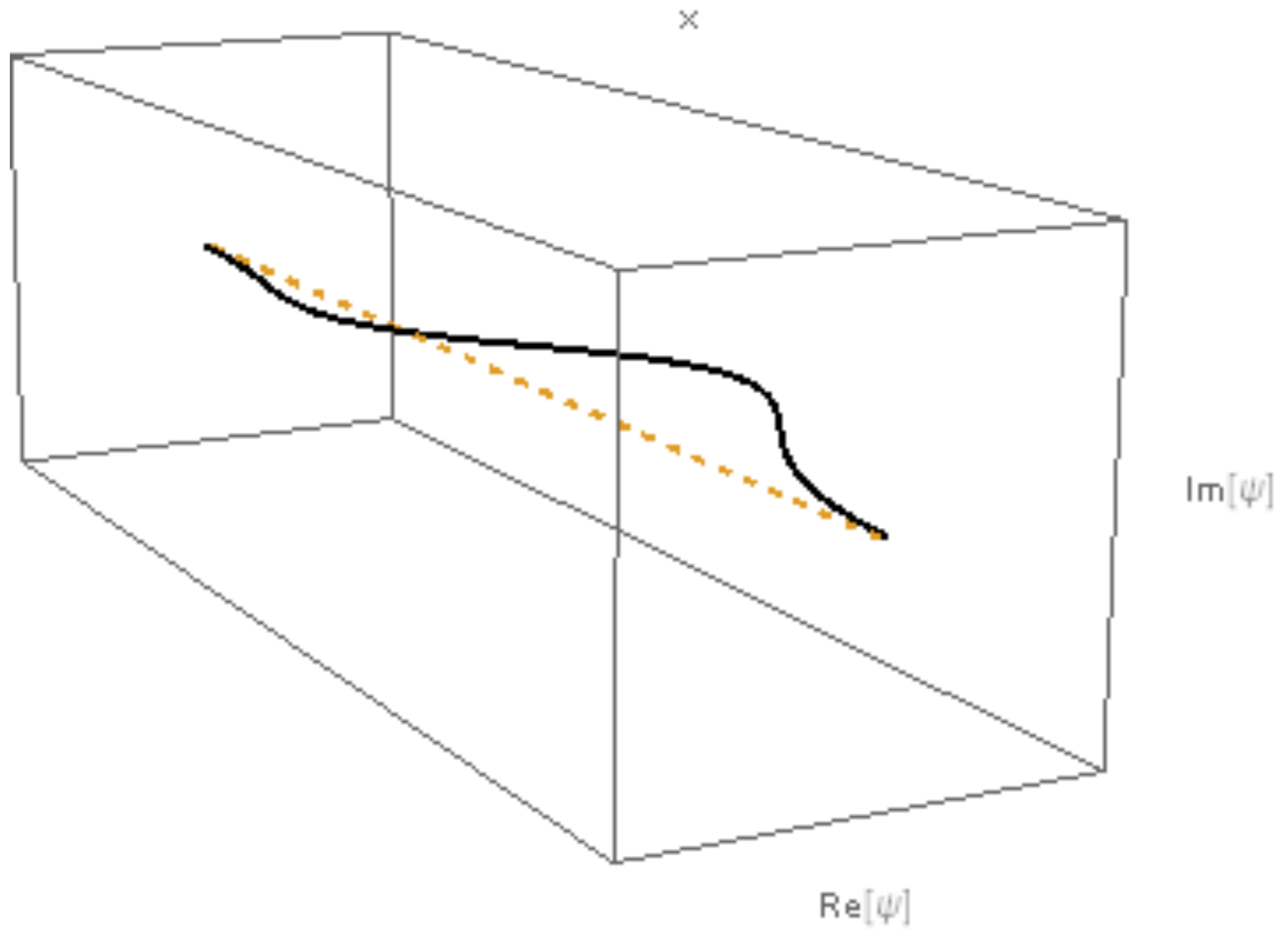
$$\psi(x) = N\left(\phi_1(x) + \phi_2(x)\right)$$

$$E_2 = 4E_1$$



Graphical Superposition

Infinite Square Well



Research about Arms

- ➡ Reasoning with Arms — Hahn, Dissertation, 2022; Frye, MS Project
- ➡ Kinesthetic Activities & Student Identity — Hahn, Dissertation, 2022
- ➡ Structural features analysis — Gire, et al., in review

Summary

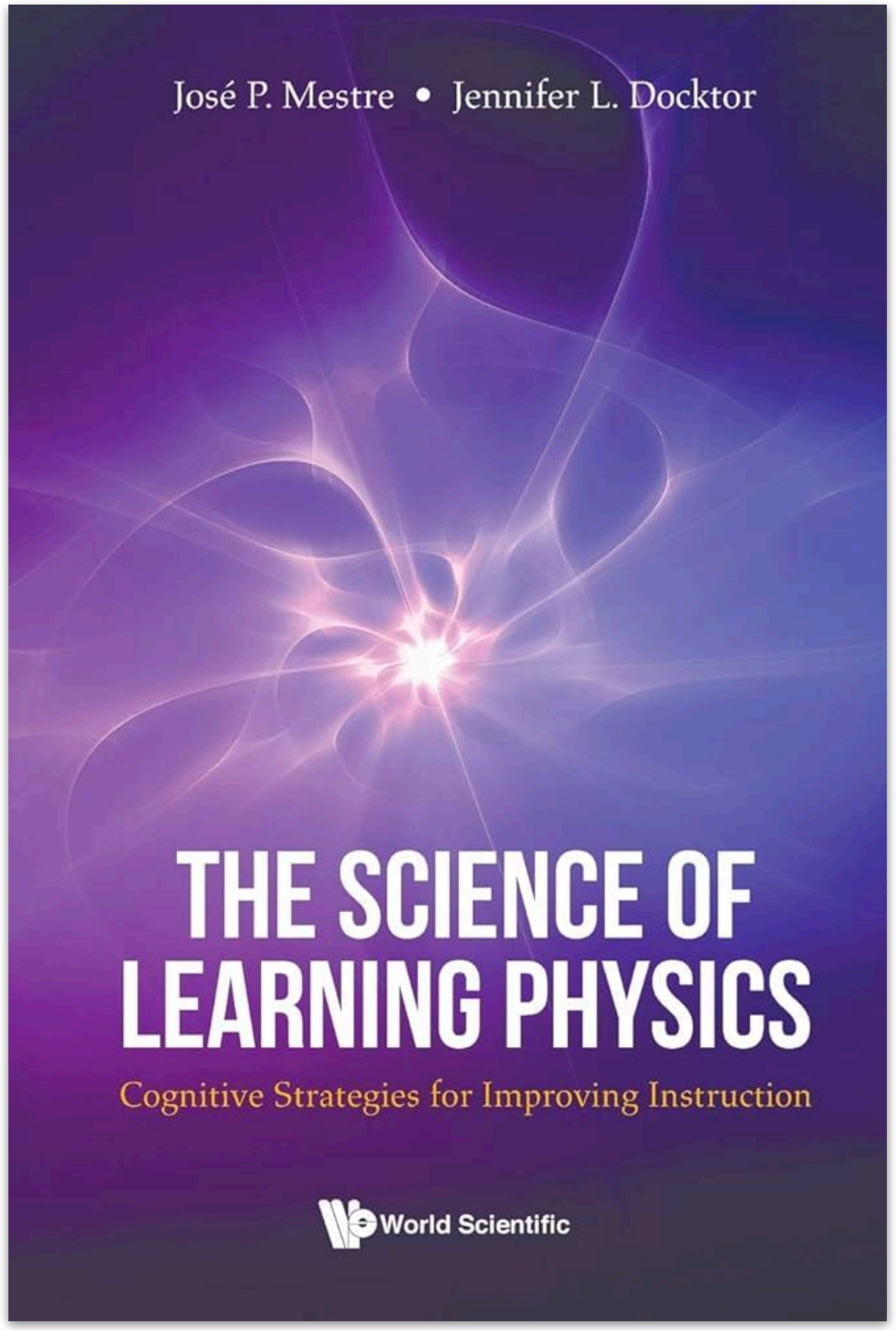
Summary

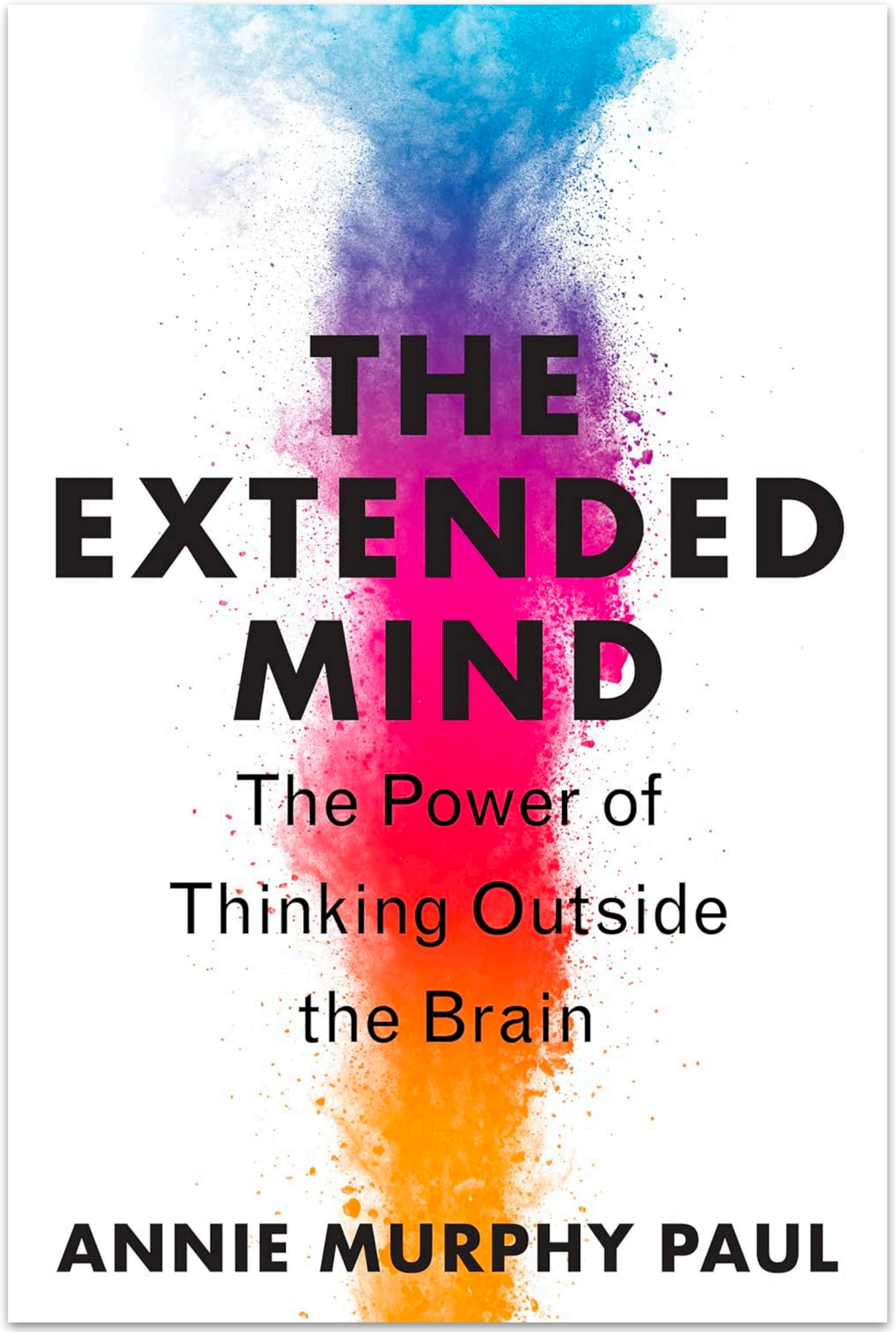
External representations are tools for doing and communicating physics

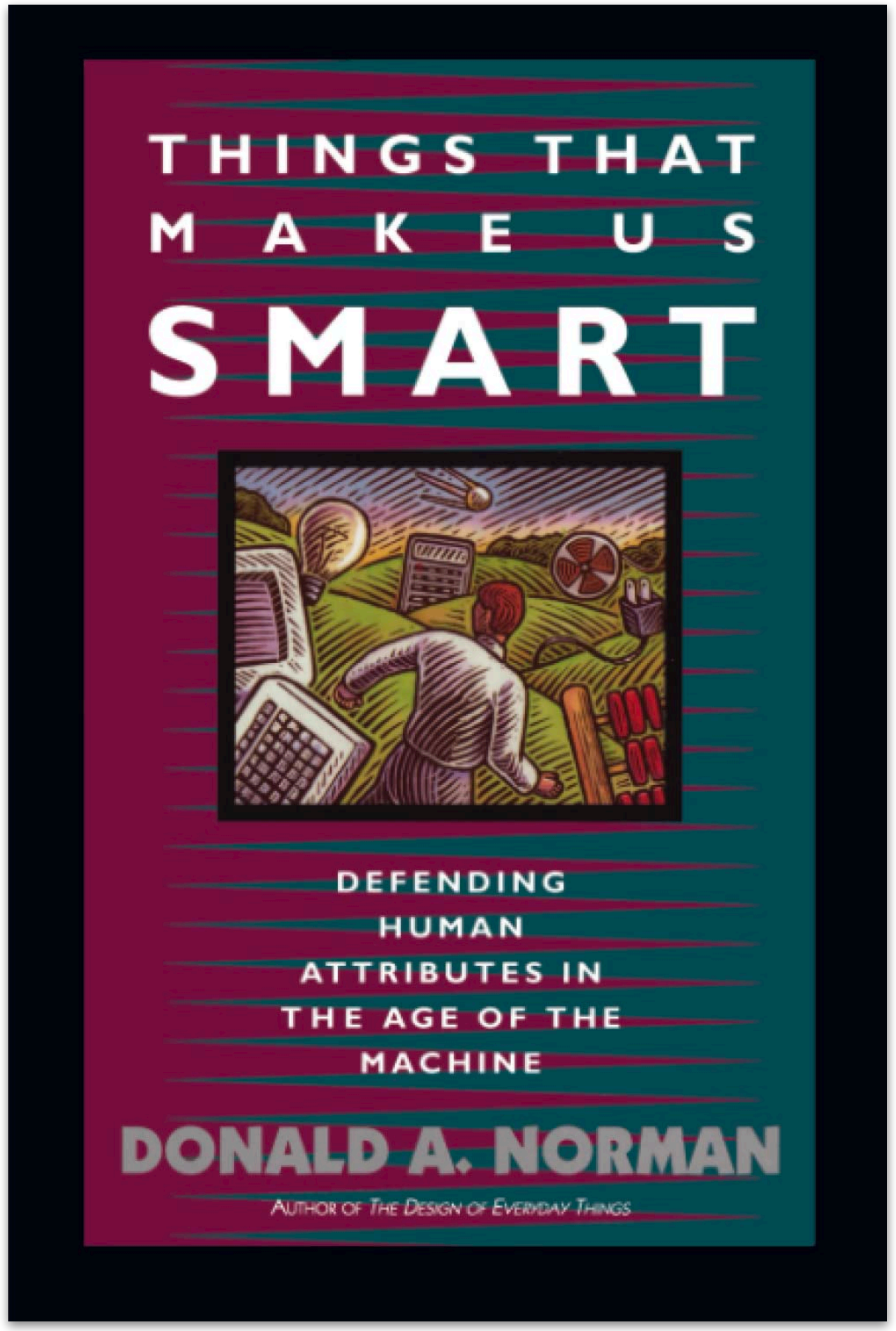
- ➡ Extra-neural resources for thinking
- ➡ Professionals & Learners need different structural features

Completeness relations, Computation, & Arms representation are promising supports for students in the transition between spin & infinite square well

3 Books Recommendations

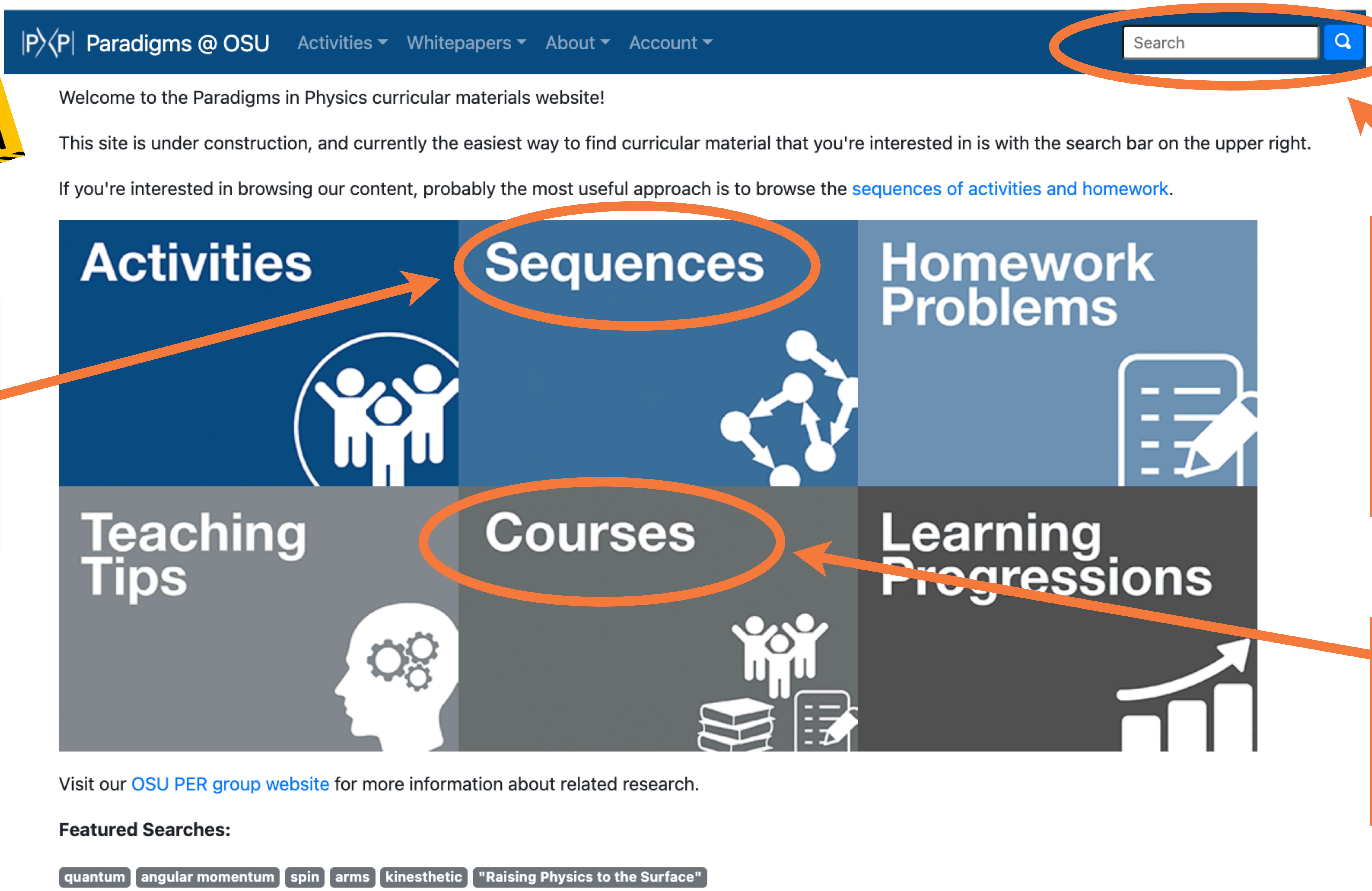






Paradigms in Physics

paradigms.oregonstate.edu



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View Whole Courses!

Thank You!

This Talk

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osuper.science.oregonstate.edu